Note #7 (Exam2 Review)

(1) Evaluate $\int \sqrt{5 + 4x - x^2} \, dx$. 
(2) Evaluate \[ \int \frac{\sqrt{9x^2 - 4}}{x^4} \, dx. \]
(3) Evaluate \( \int \frac{1}{(x^2 + 4)^2} \, dx \).
(4) Evaluate \( \int \frac{3x^2 + 4x + 3}{x^2 + 1} \, dx \).
(5) Evaluate \( \int \frac{2x^3 + 11x^2 + 18}{x^2(x^2 + 9)} \, dx \).
(6) Evaluate \[ \int \frac{x^2 + 3x}{(x - 3)(x^2 + 5)} \, dx. \]
(7) Evaluate \( \int_{0}^{\infty} \frac{\sqrt{e^{-3x} + 4}}{e^{3x}} \, dx \).
(8) Evaluate \( \int_0^e \sqrt{x \ln x} \, dx \).
(9) Compute \( \int_{0}^{4} \frac{1}{(x - 4)^3} \, dx \).
(10) Determine if the following sequences are increasing, decreasing, or not monotonic. Also, determine if each series is bounded.

(a) \( a_n = 3 - e^{2n} \)

(b) \( a_n = \frac{(-1)^n(n + 5)}{n^2 + 3n} \)
(11) Determine whether the sequence converges or diverges. If it converges, what value does it converge to?
(a) $a_n = n \sin\left(\frac{\pi}{n}\right)$
(b) $a_n = \frac{(-1)^{n-1}(7e^{2n} - 8)}{5n - 3e^{2n}}$
(12) Consider the recursive sequence \( a_1 = 4 \) and \( a_{n+1} = \frac{5}{6 - a_n} \). Assuming the sequence is decreasing and bounded, determine if the sequence converges or diverges. If it converges, find its limit.
(13) For the series \( \sum_{n=1}^{\infty} a_n \), the \( n \)th partial sum is given by \( s_n = \frac{3 - 2n}{5n + 1} \).

(a) Find \( a_n \).

(b) Does the series converge or diverge? If it converges, find its sum.
(14) Determine if the series converges or diverges.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{n(3 + \ln n)^3} \]
(b) \[ \sum_{n=1}^{\infty} \frac{3}{n^2 + 4} \]
(15) Determine if the series converges or diverges. If it converges, find its sum.

(a) \( \sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right) \)

(b) \( \sum_{n=5}^{\infty} 10\left(-\frac{2}{3}\right)^{n-1} \)
(c) \( \sum_{n=1}^{\infty} \frac{3^{n+2}}{23n+1} \)

(d) \( \sum_{n=1}^{\infty} \left[ \cos\left(\frac{1}{n+3}\right) - \cos\left(\frac{1}{n+1}\right) \right] \)
(e) \( \sum_{n=1}^{\infty} \frac{2^{n+2}}{5 \cdot 3^n} \)

(f) \( \sum_{n=1}^{\infty} \frac{n^4 + n^2}{5n - 3n^4} \)
\( (g) \sum_{n=1}^{\infty} \left[ e^{1/(n+2)} - e^{1/(n+1)} \right] \)
(16) For what values of $x$ does the series $\sum_{n=0}^{\infty} \frac{(x + 3)^{n+1}}{5^n}$ converge? Find the sum of the series for those values of $x$. 
(17) What is the smallest value of $n$ that ensures $s_n$, the $n$th partial sum, approximates the sum of the series \( \sum_{n=1}^{\infty} \frac{8}{(n+2)^3} \) with error less than 0.05?