NOTE #9 (POWER SERIES, REPRESENTATIONS OF FUNCTIONS AS POWER SERIES)

[Power Series]

(1) Find the interval and radius of convergence for the following power series.
(a) \[ \sum_{n=0}^{\infty} \frac{7^n x^n}{(2n + 1)^3} \]
(b) \[ \sum_{n=0}^{\infty} \frac{(2n)!(5x + 1)^n}{n!} \]

(c) \[ \sum_{n=0}^{\infty} \frac{n(-5)^n(x + 3)^{n+1}}{3n!} \]
(d) \[ \sum_{n=2}^{\infty} \frac{3(-1)^n(2x-5)^n}{4^{n+1} \ln n} \]
\[ (e) \sum_{n=2}^{\infty} \frac{(2x - 5)^n}{n4^n} \]
[Representations of Functions as Power Series]

(2) Find a power series representation for the function. Determine the Radius of convergence.

(a) \( f(x) = \frac{3x^3}{1 - 8x^3} \)

(b) \( f(x) = \frac{4x}{5 + 2x^2} \)
(c) $f(x) = 3x^2 \ln(9 + 2x^3)$
(d) \( f(x) = \frac{12x^5}{(4-x)^2} \)
(3) Evaluate the indefinite integral as a power series. What is the radius of convergence?

(a) \[ \int \frac{x}{2x^6 + 3} \, dx \]
(b) \( \int x^2 \arctan(3x^2) \, dx \)