Wir 3: Exam 1 Review

Sections 12.1-12.6 and 13.1-13.4

**Problem 1.** What is the equation of the sphere centered at $(6,4,12)$ with radius 6? Describe the intersection of this sphere with the three coordinate planes.

**Problem 2.** Let $a = (1, 2, -1)$ and $b = (2, -1, 2)$. Find the vector projection of $b$ onto $a$, that is $\text{proj}_a b$.

**Problem 3.** Let $a = (-2, 2, 1)$. Find a vector $b = (b_1, b_2, b_3)$ so that the scalar projection of $b$ onto $a$ equals $-4$, that is $\text{comp}_a b = -4$.

**Problem 4.** Use the figure below to answer the questions that follow.

![Diagram](image)

a.) Write $x$ in terms of $a$ and $b$.

b.) If the angle between $a$ and $b$ is $60^\circ$, $|a| = 7$, and $|b| = 6$, find $a \cdot b$.

c.) If the angle between $a$ and $b$ is $60^\circ$, $|a| = 7$, and $|b| = 6$, find $|a \times b|$ and determine whether $a \times b$ is directed into or out of the page.

**Problem 5.** Find a vector equation, a set of parametric equations, and symmetric equations for the line passing through the point $(-2,3,4)$ that is parallel to the vector $\langle 1, -4, 4 \rangle$.

**Problem 6.** Consider the line that passes through the points $(4,3,-1)$ and $(5,3,5)$. Where does this line intersect the three coordinate planes, and if it does not intersect one of the three coordinate planes, explain why not.

**Problem 7.** Find the equation of the plane that contains the point $(1,2,-5)$ and is perpendicular to the vector $\langle -6,4,-2 \rangle$.

**Problem 8.** Find parametric equations for the line that passes through $(2,-1,5)$ and is

a.) parallel to the line $\frac{x+1}{3} = \frac{y-6}{4} = z$.

b.) perpendicular to the plane $8x - 11y = 2z + 6$.

**Problem 9.** Consider the triangle with vertices $P(1,0,1)$, $Q(2,3,4)$ and $R(2,1,1)$.

a.) Find the angle at the vertex $Q$.

b.) Find the equation of the plane that passes through the points

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*Thanks to Amy Austin for generously sharing all of her WIR problems from last semester.*
Problem 10. Find the equation of the plane that passes through the point \((1, 0, 1)\) and
\(\text{a.) is perpendicular to the line } x = 9 - t, \ y = 7 + 2t, \ z = t.\)
\(\text{b.) contains line } x = 9 - t, \ y = 7 + 2t, \ z = t.\)

Problem 11. Consider the plane \(P_1\) given by the equation \(2x - y + 3z = 7\) and the plane \(P_2\) given by the equation \(3x + y + 2z = 3.\)
\(\text{a.) Find the angle between the planes.}\)
\(\text{b.) Find a point, } (x_0, y_0, z_0), \text{ that lies on both planes.}\)
\(\text{c.) Find a parametric equation for the line where the two planes intersect.}\)

Problem 12. Consider the lines \(r_1(t) = (1, 2, 0) + t(2, -2, 2)\) and \(r_2(v) = (3, 0, 2) + v(-2, 2, 0).\)
\(\text{a.) Find the point where the two lines intersect.}\)
\(\text{b.) Find an equation of the plane containing both of these lines.}\)

Problem 13. Let \(r(t) = \langle t^2, \frac{t-1}{t^2-1}, \frac{\sin t}{t} \rangle.\)
\(\text{a.) Find the domain of } r(t).\)
\(\text{b.) Find } \lim_{t \to 1} r(t).\)

Problem 14. Let \(r(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle.\)
\(\text{a.) Find the velocity and speed of the curve at time } t = \sqrt{\pi}.\)
\(\text{b.) Find } T(\sqrt{\pi}), \text{ the unit tangent vector, at } t = \sqrt{\pi}.\)
\(\text{c.) Find } a(t), \text{ the acceleration vector, at time } t.\)
\(\text{d.) The length of the curve from the point } (1, 0, 0) \text{ to the point } (1, 0, 2\pi).\)
\(\text{e.) The curvature of the curve traced out by } r(t) \text{ when } t = \sqrt{\pi}.\)

Problem 15. Find parametric equations for the tangent line to the curve \(x = 4\sqrt{t}, \ y = t^2 - 10, \ z = \frac{4}{t} \) at \((8, 6, 1)\).

Problem 16. If \(r'(t) = \langle t, e^t, te^{3t} \rangle\) and \(r(0) = \langle 1, 3, 2 \rangle\), find \(r(t).\)

Problem 17. Find \(\int_0^1 \left( \frac{4t}{t^2 + 1} \mathbf{j} - \frac{1}{1 + t^2} \mathbf{k} \right) dt.\)

Problem 18. Given the curves \(r_1(t) = \langle 3t, t^2, t^3 \rangle\) and \(r_2(v) = \langle \sin v, \sin(2v), 6v \rangle\) intersect at the origin, find the angle of intersection.

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Problem 19. Be able to match an equation with the corresponding quadric surface.

<table>
<thead>
<tr>
<th>Surface</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$</td>
<td>Cone</td>
<td>$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$</td>
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<tr>
<td></td>
<td>All traces are ellipses.</td>
<td></td>
<td>Horizontal traces are ellipses.</td>
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<tr>
<td></td>
<td>If $a = b = c$, the ellipsoid is a sphere.</td>
<td></td>
<td>Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.</td>
</tr>
<tr>
<td>Elliptic Paraboloid</td>
<td>$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$</td>
<td>Hyperboloid of One Sheet</td>
<td>$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$</td>
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<tr>
<td></td>
<td>Horizontal traces are ellipses.</td>
<td></td>
<td>Horizontal traces are ellipses.</td>
</tr>
<tr>
<td></td>
<td>Vertical traces are parabolas.</td>
<td></td>
<td>Vertical traces are hyperbolas.</td>
</tr>
<tr>
<td></td>
<td>The variable raised to the first power indicates the axis of the paraboloid.</td>
<td></td>
<td>The axis of symmetry corresponds to the variable whose coefficient is negative.</td>
</tr>
<tr>
<td>Hyperbolic Paraboloid</td>
<td>$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z^2}{c^2}$</td>
<td>Hyperboloid of Two Sheets</td>
<td>$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$</td>
</tr>
<tr>
<td></td>
<td>Horizontal traces are hyperbolas.</td>
<td></td>
<td>Horizontal traces in $z = k$ are ellipses if $k &gt; c$ or $k &lt; -c$.</td>
</tr>
<tr>
<td></td>
<td>Vertical traces are parabolas.</td>
<td></td>
<td>Vertical traces are hyperbolas.</td>
</tr>
<tr>
<td></td>
<td>The case where $c &lt; 0$ is illustrated.</td>
<td></td>
<td>The two minus signs indicate two sheets.</td>
</tr>
</tbody>
</table>

Identify the following quadric surfaces:

$$4x^2 + 9y^2 - 36z^2 = 36$$
$$16x^2 + 4y^2 + 4z^2 - 64x + 8y + 16z = 0$$
$$-4x^2 + y^2 + 16z^2 - 8x + 10y + 32z = 0$$

Thanks to Amy Austin for generously sharing all of her WIR problems from last semester.
Problem 20. Match the parametric equations with the graphs (labeled I-VI)

a. \( x = t \cos t, \ y = t, \ z = t \sin t, \ t \geq 0 \)

b. \( x = \cos t, \ y = \sin t, \ z = \frac{1}{1 + t^2} \)

c. \( x = t, \ y = \frac{1}{1 + t^2}, \ z = t^2 \)

d. \( x = \cos t, \ y = \sin t, \ z = \cos(2t) \)

e. \( x = \cos 8t, \ y = \sin 8t, \ z = e^{0.8t} \)

f. \( x = \cos^2 t, \ y = \sin^2 t, \ z = t \)

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