Wir 5: Sections 14.5, 14.6

Section 14.5

Problem 1. If \( z = \ln(9x - 6y) \), \( x = \cos(e^t) \), \( y = \sin^3(4t) \), find \( \frac{dz}{dt} \).

Problem 2. If \( w = u^2 + 2uv \), \( u = r \ln s \), \( v = 2r + s \), find \( \frac{\partial w}{\partial r} \) and \( \frac{\partial w}{\partial s} \).

Problem 3. If \( z = x^4 + xy^3 \), \( x = u^3 + w^4 \), \( y = u + ve^w \), find \( \frac{\partial z}{\partial u} \) when \( u = 1 \), \( v = 1 \), \( w = 0 \).

Problem 4. The height and radius of a right circular cone are changing with respect to time.

If the base radius of the cone is increasing at a rate of \( \frac{1}{4} \) inches per minute while the height is decreasing at a rate \( \frac{1}{10} \) inches per minute, find the rate in which the volume if the cone is changing when the radius of the cone is 2 inches and the height of the cone is 1 inch.

Problem 5. The length \( l \), width \( w \) and height \( h \) of a box change with time. At a certain instant, the dimensions are \( l = 1 \) m, \( w = 3 \) m and \( h = 2 \) m, and \( l \) and \( w \) are increasing at rate of 2 m/s while \( h \) is decreasing at a rate of 3 m/s. At that same instant, find the rate at which the surface area is changing.

Section 14.6

Problem 6. \( f(x,y) = xy \sin x \), find the directional derivative at the point \( (\frac{\pi}{2}, -1) \) in the direction \( \mathbf{u} = \left( \frac{3}{5}, \frac{4}{5} \right) \).

Problem 7. Given \( f(x,y) = x^3y^2 \), find the directional derivative at the point \( (-1, 2) \) in the direction \( 4i - 3j \).

Problem 8. If \( f(x, y) = x^2e^{xy} \), find the rate of change of \( f \) at the point \( (1, 0) \) in the direction of the point \( P(1, 0) \) to the point \( Q(5, 2) \).

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.
Problem 9. Find the gradient of \( f(x, y) = x^2 + y^2 - 4xy \) at the point \((1, -1)\).

Problem 10. If \( f(x, y) = x^2e^{-2y}, P(2, 0), Q(-3, 1) \).
   a.) Find the directional derivative at \( Q \) in the direction of \( P \).
   b.) Find a vector in the direction in which \( f \) increases most rapidly at \( P \), and find the rate of change of \( f \) in that direction.

Problem 11. Find the maximum rate of change of
\( f(x, y) = \sin^2(3x + 2y) \) at the point \( \left( \frac{\pi}{6}, -\frac{\pi}{8} \right) \) and the direction in which it occurs.

Problem 12. Find the equation of the tangent plane to the surface \( f(x, y) = x^2 + y^2 - 4xy \) at the point \((1, 2)\).

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