Wir 6: Exam 2 Review

Sections 14.1, 14.3-14.8

Problem 1. Sketch the domain of $f(x, y) = \sqrt{x^2 - y}$ and describe the level curves.

Problem 2. Sketch the domain of $f(x, y) = \ln(y^2 + x^2 - 1)$ and describe the level curves.

Problem 3. What are the level surfaces to the equation $f(x, y, x) = x + y + z$?

Problem 4. $f(x, y) = \sin(x^2 + y^2)$, find all first and second partial derivatives.

Problem 5. Find an equation for the tangent plane to the surface $z = 2x^2 + y^2$ at the point (1,1)

Problem 6. Find the tangent plane to the surface $2xy + 3yz + 7xz = -9$ at the point (1, 2, -1).

Problem 7. If $z = x^2y^2$, find the differential, $dz$, and explain what it measures.

Problem 8. Consider a rectangular box with length $l$, width $w$ and height $h$. If $A$ is the surface area of the box, find the differential, $dA$.

Problem 9. The height of a cone was measured to be 3 cm with a maximum error of 0.1 cm and the radius of the cone is measured to be 2 cm with a maximum error of 0.2 cm. Use differentials to estimate the maximum error in the calculated volume of the cone.

Problem 10. Use a linear approximation (tangent plane) to estimate $((2.1)^2 + (0.1)^3)^3$

Problem 11. Use differentials to approximate $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.
Problem 12. If \( z = e^{x^2+y^2}, \ x = e^t, \ y = \cos t, \) find \( \frac{dz}{dt} \).

Problem 13. For \( z = xy, \ x = \cos(st^2), \ y = \sin e^t, \) find \( \frac{\partial z}{\partial t} \) and \( \frac{\partial z}{\partial s} \).

Problem 14. The radius and height of a circular cylinder change with time. When the height is 1 meter and the radius is 2 meters, the radius is increasing at a rate of 4 meters per second and the height is decreasing at a rate of 2 meters per second. At that same instant, find the rate at which the volume is changing.

Problem 15. Let \( f(x,y) = \sqrt{xy} \). Find the directional derivative of \( f \) at the point \( P(4,1) \) in the direction from \( P \) to \( Q(6,2) \).

Problem 16. Let \( f(x,y) = \sqrt{xy} \). What is the direction of the largest rate of change at the point \( P(4,1) \)?

Problem 17. Let \( f(x,y) = e^{x+y} \). What is the maximum rate of change at the point \( P(-1,1) \)?

Problem 18. For the \( f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2 + 5, \) find all local minima, maxima, and saddle points.

Problem 19. Find the absolute maximum and minimum values of \( f(x,y) = 7 + xy - x - 2y \) over the closed triangular region with vertices (1,0), (5,0), (1,4).

Problem 20. Find the absolute maximum and minimum values of \( f(x,y) = 2x^3 + y^4 \) over the region \( D = \{(x,y) : x^2 + y^2 \leq 1\} \).

Problem 21. Use the method of Lagrange to find the maximum and minimum values of \( f(x,y) = 6x + 6y \) subject to the constraint \( x^2 + y^2 = 18 \).

Problem 22. Use the method of Lagrange to find the maximum and minimum values of \( f(x,y) = y^2 - x^2 \) subject to the constraint \( \frac{1}{4}x^2 + y^2 = 25 \).

Problem 23. Find the volume of the largest rectangular box with faces parallel to the coordinate planes than can be inscribed in the ellipsoid \( 16x^2 + 4y^2 + 9z^2 = 144 \).

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.