Problem 1. Identify the differential equation that corresponds to the given direction field. Based on the direction field, determine the behavior of $y$ as $t \to \infty$. If this behavior depends on the initial value of $y$ at $t = 0$, describe the dependency.

a. $y' = 2y - 1$, b. $y' = 2 + y$, c. $y' = y - 2$, d. $y' = y(y + 3)$, e. $y' = y(y - 3)$, f. $y' = 1 + 2y$, g. $y' = -2 - y$, h. $y' = y(3 - y)$, i. $y' = 1 - 2y$, j. $y' = 2 - y$
**Problem 2.** For small, slowly falling objects, the assumption that the drag force is proportional to the velocity is a good one. For larger, more rapidly falling objects, it is more accurate to assume that the drag force is proportional to the square of the velocity.

a. Write a differential equation for the velocity of a falling object of mass $m$ if the magnitude of the drag force is proportional to the square of the velocity and its direction is opposite to that of the velocity.

b. Determine the limiting velocity after a long time.

c. If $m = 10kg$, find the drag coefficient so that the limiting velocity is $49m/s$.

d. If $v(0) = 0$, find an expression for $v(t)$ at any time.

e. Find the distance $x(t)$ that the object falls in time $t$.

f. Find the time $T$ it takes the object to fall 300 m.
**Problem 3.** A radioactive material, such as the isotope thorium-234, disintegrates at a rate proportional to the amount currently present. If $Q(t)$ is the amount present at time $t$, then $dQ/dt = -rQ$, where $r > 0$ is the decay rate.

a. If 100 mg of thorium-234 decays to 82.04 mg in 1 week, determine the decay rate $r$.

b. Find an expression for the amount of thorium-234 present at any time $t$.

c. Find the time required for the thorium-234 to decay to one-half its original amount.
Problem 4. According to Newton’s law of cooling, the temperature $u(t)$ of an object satisfies the differential equation
\[
\frac{du}{dt} = -k(u - T),
\]
where $T$ is the constant ambient temperature and $k$ is a positive constant. Suppose that the initial temperature of the object is $u(0) = u_0$.

a. Find the temperature of the object at any time.

b. Let $\tau$ be the time at which the initial temperature difference $u_0 - T$ has been reduced by half. Find the relation between $k$ and $\tau$. 
Problem 5. Determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. \( t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t. \)

2. \( (1 + y^2) \frac{d^2y}{dx^2} + t \frac{dy}{dt} + y = e^t. \)

3. \( \frac{d^4y}{dt^4} + \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dt} + y = 1. \)

4. \( \frac{d^2y}{dt^2} + \sin(t + y) = \sin t. \)

5. \( 2yu_{xxx} + 4u_{xxy} + u_{yyy} = 0. \)

6. \( uu_t + u_x = 2u_{xx}. \)
Problem 6. Verify that each given function is a solution of the differential equation.

\[ t^2 y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2} \ln t. \]
Problem 7. Determine the values of $r$ for which the given differential equation has solutions of the form $y = e^{rt}$.

$$y'' + y' - 6y = 0.$$

Problem 8. Determine the values of $r$ for which the given differential equation has solutions of the form $y = t^r$ for $t > 0$.

$$t^2y'' + 4ty' + 2y = 0.$$