Problem 1. (a) Find the Laplace transform of the given function.

\[ f(t) = \int_{0}^{t} (t - \tau)^2 \cos(2\tau) d\tau \]

(b) Find the inverse Laplace transform of the given function by using the convolution theorem.

\[ F(s) = \frac{s}{(s + 1)(s^2 + 4)} \]
Problem 2. Express the solution of the given initial value problem in terms of a convolution integral.

\[ y'' + 3y' + 2y = \cos(\alpha t); \quad y(0) = 1, \quad y'(0) = 0 \]
Problem 3. Find the solutions of the differential equations.
(a) $y'' - y' - 2y = 0$

(b) $y'' + 6y' + 9y = 0$

(c) $y'' - 2y' + 5y = 0$
**Problem 4.** Find the solution of the given initial value problem using the method of undetermined coefficients.

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \ y'(0) = 1 \]
Problem 5. Find the general solution of the given differential equation using the variation of parameters.

\[ y'' - 5y' + 6y = e^{-2t} \]
Problem 6. A spring-mass system has a spring constant of 3N/m. A mass of 2kg is attached to the spring, and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity. If the system is driven by an external force of $(3 \cos(3t) - 2 \sin(3t))N$, determine the steady-state response. Express your answer in the form $R \cos(\omega t - \delta)$. 
Problem 7. Find the Laplace transform using the definition of the Laplace transform:

\[ f(t) = \begin{cases} 
1, & 0 \leq t < 5, \\
\frac{1}{t}, & 5 \leq t 
\end{cases} \]
Problem 8. Express $f(t)$ in terms of the unit step function $u_c(t)$ and find the Laplace transform.

$$f(t) = \begin{cases} 
    t, & 0 \leq t < 2, \\
    7 - t, & 2 \leq t < 5, \\
    t^2, & 5 \leq t 
\end{cases}$$
Problem 9. Find the solution of the given initial value problem.

\[ y'' + y' + \frac{5}{4}y = f(t); \quad y(0) = 0, y'(0) = 0, \]

\[ f(t) = \begin{cases} 
  t, & 0 \leq t < \frac{\pi}{2} \\
  \frac{\pi}{2}, & \frac{\pi}{2} \leq t < \infty.
\end{cases} \]