Note ♯2: Sections 2.1-2.3

Problem 1. Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

$$ty' + y = 3t\cos(2t), \quad t > 0$$

Problem 2. Find the solution of the given initial value problem.

$$ty' + (t+1)y = t, \quad y(\ln 2) = 1, \quad t > 0$$

Problem 3. Solve the initial value problem and describe the behavior of the solutions corresponding to the initial value a. Let a_0 be the value of a for which the solutions transition from one type of behavior to another. Find a_0.

$$3y' - 2y = e^{-\pi t/2}, \quad y(0) = a$$

Problem 4. Solve the given differential equation (express the solution in implicit form).

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$$

Problem 5. Find the solution of the given initial value problem in explicit form.

$$y' = (3x^2 - e^x)/(2y - 5), \quad y(0) = 1$$

Problem 6. a. Find the solution of the given initial value problem in explicit form.

b. Determine (at least approximately) the interval in which the solution is defined.

$$y' = 2y^2 + xy^2, \quad y(0) = 1.$$

Problem 7. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

Problem 8. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

Problem 9. Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.

a. Find the time T required for the original sum to double in value as a function of r.

b. Determine T if r = 7%.

c. Find the return rate that must be achieved if the initial investment is to double in 8 years.

Problem 10. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of $200^o F$ when freshly poured, and 1 min later has cooled to $190^o F$ in a room at $70^o F$, determine when the coffee reaches a temperature of $150^o F$.