Note #2: Sections 2.1-2.3

Problem 1. Find the general solution of the given differential equation, and use it to determine how solutions behave as \( t \to \infty \).

\[ ty' + y = 3t \cos(2t), \quad t > 0 \]
Problem 2. Find the solution of the given initial value problem.

\[ ty' + (t + 1)y = t, \quad y(\ln 2) = 1, \quad t > 0 \]
Problem 3. Solve the initial value problem and describe the behavior of the solutions corresponding to the initial value \( a \). Let \( a_0 \) be the value of \( a \) for which the solutions transition from one type of behavior to another. Find \( a_0 \).

\[ 3y' - 2y = e^{-\pi t/2}, \quad y(0) = a \]
Problem 4. Solve the given differential equation (express the solution in implicit form).

\[ \frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y} \]
Problem 5. Find the solution of the given initial value problem in explicit form.

\[ y' = \frac{(3x^2 - e^x)}{(2y - 5)}, \quad y(0) = 1 \]
Problem 6. a. Find the solution of the given initial value problem in explicit form.

b. Determine (at least approximately) the interval in which the solution is defined.

\[ y' = 2y^2 + xy^2, \quad y(0) = 1. \]
Problem 7. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.
Problem 8. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.
Problem 9. Suppose that a sum $S_0$ is invested at an annual rate of return $r$ compounded continuously.

a. Find the time $T$ required for the original sum to double in value as a function of $r$.

b. Determine $T$ if $r = 7\%$.

c. Find the return rate that must be achieved if the initial investment is to double in 8 years.
Problem 10. Newton’s law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton’s law of cooling. If the coffee has a temperature of 200°F when freshly poured, and 1 min later has cooled to 190°F in a room at 70°F, determine when the coffee reaches a temperature of 150°F.