NOTE ♯2: SECTIONS 2.4-2.5

Problem 1. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

a.

$$(4 - t^2)y' + 2ty = 3t^2, \quad y(-3) = 1$$

b.

$$(t - 3)y' + (\ln t)y = 2t, \quad y(1) = 2$$

Problem 2. State where in the ty-plane the hypotheses of Theorem 2.4.2(existence and uniqueness theorem for nonlinear equations) are satisfied.

a.

$$y' = (t^2 + y^2)^{3/2}$$

b.

$$y' = \frac{1 + t^2}{3y - y^2}$$

Problem 3. Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0.

a.

$$y' + y^3 = 0, \qquad y(0) = y_0$$

b.

$$y' = \frac{t^2}{y(1+t^3)}, \qquad y(0) = y_0$$

Problem 4.

$$dy/dt = (y-4)(y-2)(y+1)$$

a. Determine the critical (equilibrium) points.
b. Sketch the graph of $f(y)$ versus y.
c. Draw the phase line.
d. Classify equilibrium points.
e. Sketch several graphs of solutions in the ty-plane.

Problem 5.

$$dy/dt = (y-3)^2(y-1)(y+2)^2$$

a. Determine the critical (equilibrium) points.
b. Draw the phase line.
c. Classify equilibrium points.
d. Sketch several graphs of solutions in the ty-plane.

Problem 6. Another equation that has been used to model population growth is the Gompertz equation

$$dy/dt = ry \ln \frac{K}{y},$$

where r and K are positive constants.

a. Sketch the graph of $f(y)$ versus y, find the critical points, and determine whether each is asymptotically stable or unstable.

b. For $0 \leq y \leq K$, determine where the graph of y versus t is concave up and where it is concave down.

c. Solve the Gompertz equation subject to the initial condition $y(0) = y_0$. Hint: You may wish to let $u = \ln(y/K)$.