



WEEK IN REVIEW #1

1. Find the general solution of the given differential equation.

(a) $y' + 2ty = 2te^{-t^2}$

(b) $2\sqrt{x}y' = \sqrt{1-y^2}$



(c) $ty' + y = 3t \cos t, \quad t > 0.$

2. Find the solution to the initial value problem and the interval of validity in each case.

(a) $2\sqrt{x} \frac{dy}{dx} = \cos^2 y, \quad y(4) = \frac{\pi}{4}.$



(b) $\frac{dy}{dt} + \frac{2y}{t} = \frac{\cos t}{t^2}$ $y(1) = \frac{1}{2}$, $t > 0$.



3. Consider the initial value problem

$$y' + 2y = 5 - t, \quad y(0) = y_0$$

Find the value y_0 for which the solution touches, but does not cross the t -axis.



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4. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gallons of water. Brine containing 2 lb/gal of salt flows into the tank at a rate of 4 gal/min, and the well-stirred mixture flows out of the tank at a rate of 3 gal/min. How much salt does the tank contain when it is full?



5. Consider a large lake formed by damming a river that initially holds 200 million liters of water. Because a nearby chemical plant uses the lake's water to clean its reservoirs, 1,000 liters of brine, each containing $100(1 + \cos t)$ kilograms of dissolved pollution, run into the lake every hour. Let's make the simplifying assumption that the mixture, kept uniform by stirring, runs out at the rate of 1,000 liters per hour and no additional spraying causes the lake to become even more contaminated. Find the amount of pollution $p(t)$ in the lake at any time t , and determine its variations in the long run.



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6. A cake is removed from an oven at 210° F and left to cool at room temperature, which is 70° F. After 30 min the temperature of the cake is 140° F. When will it be 100° F?



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7. A ball with mass 1kg is thrown upward with initial velocity 20 m/s from the roof of a building 50 m high. A force due to the resistance of the air of $v/10$, where the velocity is measured in m/s, acts on the ball. Find the maximum height above the ground that the ball reaches



8. In a typical Swedish summer day, temperature fluctuates sinusoidally between 10 (at midnight) and 30 (at noon) degrees Celsius. Assume at a party someone forgets a beer with a temperature of 5 degrees at 10 pm, but finds it again at 2 am. What will be the temperature of the beer then, assuming a heat transfer coefficient of 0.1 per hour.