1. Determine an interval in which the solution of the following initial value problem is certain to exist.

\[(t^2 - 1)y' + (\sin t)y = \frac{\cot t}{t^2 - 4t + 3}, \quad y(2) = -1\]

2. State where in the \(ty\)-plane the hypothesis of the Existence and Uniqueness theorem are satisfied for the following differential equations.

(a) \[y' = \frac{\ln(ty)}{1-(t^2+y^2)}\]

(b) \[y' = (t^2 - y)^{1/3}\]
3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value $y_0$.

(a) $y' = -\frac{4}{t}y$, $y(2) = y_0$

(b) $y' + y^3 = 0$, $y(t_0) = y_0$
4. Verify that both $y_1 = 1 - t$ and $y_2 = \frac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t + \sqrt{t^2 + 4y}}{2}, \quad y(2) = -1.$$ 

Does it contradict the existence and uniqueness theorem?
5. Given the differential equation

\[ y' = y^3 - 4y \]

(a) Find the equilibrium solutions.
(b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.
(c) Graph some solutions.
(d) If \( y(t) \) is the solution of the equation satisfying the initial condition \( y(0) = y_0 \), where \( -\infty < y_0 < \infty \), find the limit of \( y(t) \) when \( t \) increases.
6. Given the differential equation

\[ y'(t) = y^3 - 2y^2 + y \]

(a) Find the equilibrium solutions.
(b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semistable.
(c) Sketch the graph of some solutions.
(d) If \( y(t) \) is the solution of the equation satisfying the initial condition \( y(0) = y_0 \), where \(-\infty < y_0 < \infty\), determine the behavior of \( y(t) \) as \( t \) increases.
(e) Do any solutions ‘blow up in finite time,’ namely, do they admit a vertical asymptote?