Week in Review ♯3

1. Determine an interval in which the solution of the following initial value problem is certain to exist.

$$(t^2-1)y' + (\sin t)y = \frac{\cot t}{t^2-4t+3}, \qquad y(2) = -1$$

2. State where in the ty-plane the hypothesis of the Existence and Uniqueness theorem are satisfied for the following differential equations.

   (a) $y' = \frac{\ln(ty)}{1-(t^2+y^2)}$

   (b) $y' = (t^2-y)^{1/3}$.

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value y_0.
   (a) $y' = \frac{-4}{t} y, \qquad y(2) = y_0$

   (b) $y' + y^3 = 0 \qquad y(t_0) = y_0$

4. Verify that both $y_1 = 1 - t$ and $y_2 = \frac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t + \sqrt{t^2 + 4y}}{2}, \qquad y(2) = -1.$$

Does it contradict the existence and uniqueness theorem?

5. Given the differential equation

$$y' = y^3 - 4y$$

   (a) Find the equilibrium solutions.
   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.
   (c) Graph some solutions.
   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when t increases.

6. Given the differential equation

$$y'(t) = y^3 - 2y^2 + y$$

   (a) Find the equilibrium solutions.
   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semistable.
   (c) Sketch the graph of some solutions.
   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, determine the behavior of $y(t)$ as t increases.
   (e) Do any solutions 'blow up in finite time,' namely, do they admit a vertical asymptote?