Week in Review ♯4

1. Determine if the equations are exact and solve the ones that are:
   (a) $(2x + 5y)dx + (5x - 6y)dy = 0.$

   (b) $1 + \dfrac{y}{x} - \frac{1}{x}y' = 0$

(c) $(\sin(2t) + 2y)dy + (2y\cos(2t) - 6t^2)dt = 0$

2. Show that the following equations are not exact. However, if you multiply by the given integrating factor, the resulting equation is exact.
   (a) $(x^2 + y^2 - x)dx - ydy = 0 \qquad \mu(x, y) = \frac{1}{x^2+y^2}$

(b) $3(y+1)dx - 2xdy = 0, \qquad \mu(x,y) = \frac{y+1}{x^4}$

3. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

and then solve the equation.

4. Solve the IVP $(3x^2 + 2xy^2)dx + 2x^2y dy = 0,\ y(2) = -3$.

5. Solve $(x^4 \ln x - 2xy^3)dx + 3x^2y^2 dy = 0$.

6. Solve $y' = \dfrac{(1+x)e^x}{xe^x - ye^y}$.

7. Solve
$$ydx = (y^2 + x^2 + x)dy,$$
if we know there is an integrating factor of the form $\mu = \phi(x^2 + y^2)$.