Week in Review §7

1. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2 \cos 3t$ lb,

(a) Formulate the initial value problem describing the motion of mass

(b) Solve the initial value problem.

(c) If the given external force is replaced by a force $4 \cos \omega t$ of frequency $\omega$, find the value of $\omega$ for which resonance occurs.
2. A 3 kg object is attached to a spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form $F(t) = 10 \cos(\omega t)$ is attached to the object and the system will experience resonance. If the object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward find the displacement at any time $t$. 
3. Use the definition to find the Laplace transforms of
   (a) \( f(t) = e^{at} \) where \( a \) is a non zero real number.

   (b) \( f(t) = \sin(bt) \) where \( b \) is a non zero real number.
(c) \[ f(t) = \begin{cases} 5 - t & 0 \leq t < 2 \\ 3t & 2 \leq t. \end{cases} \]
(d) \[ f(t) = t \]

(e) \[ f(t) = t^2 \]