



SECTION 1.1: BASIC MATRIX OPERATIONS

- Size (Dimensions): rows  $\times$  columns
- Entries: Labeled based on row and column position,  $a_{ij}$
- Addition/Subtraction:
  - Matrices must be the same size for the operation to be performed
  - Combined corresponding entries based on operation given
- Scalar Product: multiplying a matrix by a constant results in a matrix of the same size
- Transpose of a matrix  $A$ :  $A^T$
- Matrix Equality: two matrices are equal if they are the same size AND corresponding entries are equal
- Operations of matrices which contain variables must be done by hand

**Pr 1.** Use the given information about the sizes of matrices  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  to determine the size of the resulting matrix, if the computation is possible. If the product is not possible, explain why.

$A$  is a  $1 \times 2$ ,       $B$  is a  $1 \times 2$ ,       $C$  is a  $2 \times 3$ ,       $D$  is a  $2 \times 3$        $E$  is a  $3 \times 2$

a.  $\frac{2}{3}B$

b.  $(B + C)^T$

c.  $D - D$

d.  $E^T - D + 2C$

**Pr 2.** Given matrices  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  below, determine each part.

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix}$$

$$E = \begin{bmatrix} v & 81 \\ 4y & (42-w) \end{bmatrix}$$

- State the dimensions of each matrix.
- If it exists, state the value of  $b_{23}$ .
- If it exists, state the value of  $c_{43}$ .

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix}$$

$$E = \begin{bmatrix} v & 81 \\ 4y & (42-w) \end{bmatrix}$$

d. If  $M = B^T$ , state the value of  $m_{31}$ , if it exists.

e. Compute  $A + B$ , if possible. If the operation is not possible, explain why.

f. Compute  $C^T - 4A + 0.5B$ , if possible. If the operation is not possible, explain why.

g. Compute  $\frac{3}{2}D + 4C$ , if possible. If the operation is not possible, explain why.

h. Compute  $2E - 3D$ , if possible. If the operation is not possible, explain why.

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix}$$

$$E = \begin{bmatrix} v & 81 \\ 4y & (42-w) \end{bmatrix}$$

i. Compute  $(B^T + C)^T$ , if possible. If the operation is not possible, explain why.

j. Compute  $-2A^T - E^T$ , if possible. If the operation is not possible, explain why.

k. If  $A + B = C^T$ , determine the values of  $w$ ,  $x$ , and  $y$ .

l. For what values of  $v$ ,  $w$ ,  $x$ , and  $y$  is  $D = 2E$ ?

**Pr 3.** Solve the matrix equation for matrix  $X$ .

$$8X - \begin{bmatrix} 120 & 250 \\ 480 & 275 \end{bmatrix} = \begin{bmatrix} 270 & -140 \\ -20 & 225 \end{bmatrix} + 3X$$

**Pr 4.** Solve the matrix equation,  $B + 4X = 2E - 6X$ , for matrix  $X$ , assume that all matrices,  $B$ ,  $X$ , and  $E$  are the same size.

**Pr 5.** An online streaming service records the number of downloads based genre and media type. During the month of January 3000 science fiction novels, 4500 science fiction movies, 4200 thriller novels, 7000 thriller movies, 1200 romance novels, and 6800 romantic movies were downloaded, while in February the downloads were 3800, 2900, 2600, 5100, 4500, and 9700 respectively.

a. Write a  $3 \times 2$  matrix,  $J$ , representing the online streaming service data in January.

b. Write a  $3 \times 2$  matrix,  $F$ , representing the online streaming service data in February.

c. Write a matrix operation that would represent the change in sales from January to February.

d. Explain in the context of the application what entry  $c_{12}$  tells us, if  $C = J + F$ .

e. If the streaming service expects a 30% increase in all sales from February to March, how many romantic movies are expected to be downloaded?

SECTION 1.2: MATRIX MULTIPLICATION

- For the matrix product  $AB$  to exist the number of columns of matrix  $A$  must be the same as the number of rows of matrix  $B$ .
- Matrix multiplication is not commutative.

**Pr 1.** Use the given information about the sizes of matrices  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  to determine the size of the resulting matrix, if the computation is possible. If the product is not possible, explain why.

$A$  is a  $1 \times 3$ ,  $B$  is a  $2 \times 2$ ,  $C$  is a  $2 \times 4$ ,  $D$  is a  $3 \times 3$ ,  $E$  is a  $4 \times 3$ ,  $F$  is a  $2 \times 2$

a.  $AD$

b.  $BC$

c.  $CB$

d.  $EA^T$

e.  $7CC^T$

$A$  is a  $1 \times 3$ ,  $B$  is a  $2 \times 2$ ,  $C$  is a  $2 \times 4$ ,  $D$  is a  $3 \times 3$ ,  $E$  is a  $4 \times 3$ ,  $F$  is a  $2 \times 2$

f.  $FCE$

g.  $DAC$

h.  $AE^T C^T$

i.  $(F + B)C$

j.  $C^T(2B - 3F)$