Section 6.1: Interest and Effective Rates

- Simple Interest - $I = Prt$ and $A = P(1 + rt)$
- Compound Interest - TVM Solver
- Compounded Continuously Interest - $A = P e^{rt}$
- Effective Interest Rate $r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$ and $r_{eff} = e^r - 1$

Pr 1. You invest $600 in an account paying simple interest at a rate of 8% per year.

(a) How much interest does the account earn in two years?

\[ I = Prt \]
\[ I = 600 \times (0.08) \times 2 \]
\[ I = 96 \]

The account earns $96 in interest in 2 yrs.

(b) How much money is in the account at the end of the two years?

\[ A = P + I \]
\[ A = 600 + 96 \]
\[ A = 696 \]

After two years there is $696 in the account.

Pr 2. You borrow $5000 from a payday loan company and when you pay back the loan you owe $7500. How long did you borrow the money for, if the payday loan charges a simple interest rate of 67% per year?

\[ A = 7500 \]
\[ P = 5000 \]
\[ r = 0.67 \]
\[ t = ? \]

\[ A = P \left(1 + rt\right) \]
\[ 7500 = 5000 \left(1 + 0.67t\right) \]
\[ \frac{7}{5} = 1 + 0.67t \]
\[ \frac{2}{5} = 0.67t \]
\[ t \approx 0.75 \text{ years} \]

you borrowed the money for 0.75 yrs.

Pr 3. Determine the annual simple interest rate at which $1500 will grow to $1580 in 7 months? Round your answer to two decimal places?

\[ I = 1580 - 1500 = 80 \]
\[ I = Prt \]
\[ 80 = 1500 \left(r\right) \left(\frac{3}{12}\right) \]
\[ 80 = 875r \]
\[ \frac{80}{875} = r \approx 0.0914 \]
\[ r \approx 9.14\% \text{ annual simple interest} \]
Pr 4. You put $2,000 into an account and 5 years later have $8,450.50 in the account. If the account earned interest compounded monthly, what was the account’s annual interest rate? Round your answer to two decimal places?

\[
\begin{align*}
N &= \frac{t \times 12}{m} = 12 \cdot 5 \\
I% &= ? \quad \rightarrow \quad 29.17047419 \\
PV &= -2000 \\
PMT &= 0 \\
FV &= 8450.50 \\
P/Y &= m = 12 \\
C/Y &= m = 12 \\
PMT: \text{ END}
\end{align*}
\]

I% is 29.17%.

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Pr 5. You want to take a trip in 3 years that will cost $18,000. How much should you deposit now, making no additional deposits, into an account that earns 8% per year, compounded daily, so you will have enough money for the trip?

\[
\begin{align*}
N &= 3 \times 365 = 1095 \\
I% &= 8 \\
PV &= ? \quad \rightarrow \quad -14159.67386 \\
PMT &= 0 \\
FV &= 18000 \\
P/Y &= 365 \\
C/Y &= 365 \\
PMT: \text{ END}
\end{align*}
\]

You need to invest $14,159.67.
Pr 6. You invest $12000 into an account that pays annual interest at a rate of 5.96%, compounded monthly.

\[ N = 12 \cdot (4/12) \]
\[ I\% = 5.96 \]
\[ PV = -12000 \]
\[ PMT = 0 \]
\[ FV = ? \rightarrow 12240.18197 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: \text{END} \]

(a) How much money will you have in the account at the end of 4 months? Assuming no additional deposits are made.

$12,240.18

(b) How much interest did the account earn in the 4 months?

\[
\text{Interest Earned} = \text{Ending Balance - Starting Balance} = A - P = 12240.18 - 12000 = 240.18
\]

Pr 7. You invest $12000 into an account that pays annual interest at a rate of 5.96% per year, compounded continuously. How much money is in the account at the end of 4 months?

\[ A = Pe^{rt} \]
\[ A = 12000 \cdot e^{0.0596 \cdot (4/12)} \]
\[ A = 12240.78387 \]

The account has $12,240.78.

Pr 8. What is the annual interest rate, as a percent, on an account that is compounded continuously, if $2000 grows to $3500 in 4 years? Round your answer to two decimal places.

\[ 3500 = 2000 \cdot e^{0} \]
\[ 7/4 = e^{0} \]
\[ \ln (7/4) = 0 \]
\[ \ln (7/4) = 4r \]
\[ r = \frac{\ln (7/4)}{4} \approx 0.13990 \approx 13.99\% \]
Pr 9. While binge watching Magnum P.I. reruns (that mustache is mesmerizing) you see ads for accounts from three different banks. Bank A advertises a nominal rate of 7.15% per year, compounded semi-annually. Bank B advertises a nominal rate of 7% per year, compounded daily. Bank C advertises an APR of 6.95%, compounded continuously.

(a) What is the effective rate for each account?

\[ \text{A: Eff}(7.15, 2) = 7.2778\% = \text{APY} \]

\[ \text{B: Eff}(7, 365) = 7.2501\% = \text{APY} \]

\[ \text{C: } (\text{e}^{0.0695} - 1)100 = 7.1972\% = \text{APY} \]

(b) Which bank has the best interest rate for an investment?

Account A is best, it has the largest APY, you will earn more money in interest.

(c) If you were to borrow money instead, which account would be the best for you?

Account C is best, it has the smallest APY, you owe less in interest.
Section 6.2: Annuities, Sinking Funds, and Amortization

- Ordinary Annuities
- Sinking Funds
- Loans
  - Down Payment
  - Total Interest
  - Outstanding Principal/Balance
  - Equity
- Finance Applications

Pr 1. You would like to save up $70,000 over the next 15 years. You have $900 that you will use as an initial deposit, and then will make quarterly payments for the next 15 years. The account has an interest rate of 7.5% per year, compounded quarterly. How much should you deposit each quarter in order to reach your goal?

\[
\begin{align*}
N &= 4 \cdot 15 \\
I\% &= 7.5 \\
PV &= 900 \\
PMT &= ? \rightarrow 615.66240 \\
FV &= 70000 \\
P/Y &= 4 \\
C/Y &= 4 \\
PMT: \text{ END}
\end{align*}
\]

Each quarter you should deposit $615.66

Pr 2. Your parents started saving account, for your college expenses, when you turned three years old. They place $100 into the account each month. If the account has an annual interest rate of 1.6%, compounded monthly.

\[
\begin{align*}
N &= 12 \cdot 15 \\
I\% &= 1.6 \\
PV &= 0 \quad \text{(no mention)} \\
PMT &= -100 \\
FV &= ? \rightarrow 20328.44605 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT: \text{ END}
\end{align*}
\]

(a) How much money will be in the account when you turn 18? \(18 - 3 = 15\)

\[520,328.45\]

(b) How much did your parents invest in the account?

\[
\text{Total Investment} = PV + N \cdot (PMT) = 0 + 180(100) = 18,000
\]

They invested $18,000
Pr 3. You have deposited $125 in your IRA at the end of each month for the past 30 years. Your investment has earned an fixed APR of 8%, compounded monthly, over the entire 30 years. Now, at age 55, you are considering retirement. If you keep the money in the account and it maintains a fixed APR of 8%, compounding monthly, what monthly payment will you receive over the next 30 years?

\[ N = 12 \times 30 \]
\[ I\% = 8\% \]
\[ PV = 0 \]
\[ PMT = -125 \]
\[ FV = ? \rightarrow 186,294.93 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: \text{END} \]

Your monthly payment is $1366.97.

Pr 4. You owe $4500 on a credit card that charges 14.5% per year, compounded monthly. If you only pay the minimum amount of $60 per month, how long will it take you to pay off the credit card, if you make no additional purchases on the card?

\[ N = 12 \times t \rightarrow 197.0810778 \]
\[ I\% = 14.5\% \]
\[ PV = 4500 \]
\[ PMT = -60 \]
\[ FV = 0 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: \text{END} \]

12 \times t = 198

\[ t = \frac{198}{12} \]

\[ t = 16.5 \text{ years} \]
Pr 5. The Kleins have decided to buy a house. They can make a down payment of $30,000 and monthly payments up to $800. The lowest rate that they were quoted was 7.2% per year, compounded monthly, for 30 years.

(a) What is the most expensive house they can afford to buy?

\[
\begin{align*}
N &= 12 \times 30 \\
I\% &= 7.2 \\
PV &= \_ \rightarrow 1757,0854 \\
PMT &= -800 \\
FV &= 0 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT: \text{END}
\end{align*}
\]

Loan Amount = $117,857.09

Down Payment = $30,000.00

Purchase Price = $147,857.09

(b) Suppose the house that they decide to buy has a price tag of $139,000. What are the monthly payments that they would pay to amortize(payoff) the loan?

\[
\begin{align*}
N &= 12 \times 30 \\
I\% &= 7.2 \\
PV &= 109,000 \\
PMT &= ? \rightarrow 739.87914 \\
FV &= 0 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT: \text{END}
\end{align*}
\]

Their monthly payment will be $739.88.
Pr 6. The Phredds is buying some land for a price of $224,000. They make a 15% down payment and borrow the rest from a bank at an interest rate of 3.52% per year, compounded monthly. The loan will have to be paid off in 15 years.

(a) What are their monthly mortgage payments be?

\[ N = 12 \cdot 15 = 180 \]
\[ I\% = 3.52 \]
\[ PV = 224000 \cdot .85 = 190,400 \]
\[ PMT = \] → \$1363.01 \]
\[ FV = 0 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: END \]

Monthly Payment is \$1363.01

(b) What is the outstanding balance on the loan after seven years?

\[ N = 12 \cdot 7 = 84 \]
\[ I\% = 3.52 \]
\[ PV = 190,400 \]
\[ PMT = -1363.01 \]
\[ FV = ? \] → \$113,895.02 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: END \]

\$113,895

(c) After seven years of making payments, how much equity will they have in the land? Assume the value of the land remains constant.

Equity = Purchase Price - Outstanding Balance
= 224000 - 113 895
= \$110,105

(d) How much interest did the Phredds pay the bank?

Total interest = What paid into loan - Loan amount
= N(PMT) - PV
= 180(1363.01) - 190,400

Interest = \$54,941.80
Pr 7. You currently owe $4,500 to the store for the furniture that you purchased. You made a down payment of $3,000 and have been making payments of $350 each month for the last three years. The store is charging you interest of 6.3% APR, compounded monthly, on the loan.

\[
\begin{align*}
N &= 12 \cdot 3 \\
I\% &= 6.3 \\
PV &= ? \rightarrow 15,180.49 \\
PMT &= -350 \\
FV &= -4500 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT: END
\end{align*}
\]

Loan Amount is $15,180.49

(a) What was the purchase price of the furniture?

\[
\text{Purchase Price} = \text{Down Payment} + \text{Loan Amount} = 3000 + 15,180.49 = 18,180.49
\]

(b) How many more payments will you have to make until the furniture is paid off?

\[
\begin{align*}
N &= 12 \cdot t \rightarrow 49.3465 \text{ meaning a total of 50 payments is needed.} \\
I\% &= 6.3 \\
PV &= 15,180.49 \\
PMT &= -350 \\
FV &= 0 \\
P/Y &= 12 \\
C/Y &= 12 \\
PMT: END
\end{align*}
\]

But we already made 12 \cdot 3 = 36 payments so far.

50 - 36 = remaining payments

14 more payments
Pr 8. If you buy a television set for $2500 and agree to pay for it in eighteen equal monthly payments with an annual interest rate of 18%, compounded monthly, how much are your monthly payments?

\[ N = 18 \]
\[ I\% = 18 \]
\[ PV = 2500 \]
\[ PMT = ? \rightarrow -159.51 \]
\[ FV = 0 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ PMT: \text{END} \]

(a) How much are your monthly payments?

\[ $159.51 \]

(b) How much of the first payment goes towards interest?

\[ PV = 2500 \quad 2500 \left(0.18 \over 12\right) = 37.50 \]

(c) How much of the 10th payment will go towards the balance?

What is the previous balance after 9 payments?

\[ N = 18 - 9 = 9 \text{ payments left} \]
\[ PMT = -159.51 \]
\[ FV = 0 \]
\[ PV = ? \rightarrow 1333.47 \]
\[ P/Y = 12 \]
\[ C/Y = 12 \]
\[ END = \]
2) How much of 10th payment goes to interest
\((1333.59)(\frac{0.15}{12})\)
= 20.00
\text{to interest}

\(1333.57 \left( \frac{0.15}{12} \right)\)
= 20.01
\text{to interest}

3) What goes toward the balance is
Payment - what goes to interest

\(159.51 - 20.00 = \boxed{139.51}\)

\(159.51 - 20.01 \approx \boxed{139.50}\)