Section 2.2: Modeling with Linear Functions

- Linear Depreciation, $V(t) = mt + b$
- Cost, variable cost + fixed costs $C(x) = mx + F$
- Revenue, price per item times quantity sold $R(x) = px$
- Profit, revenue minus cost $P(x) = R(x) - C(x)$
- Demand, $D(x) =$ price $= mx + b$, consumer point of view
- Supply, $S(x) =$ price $= mx + b$, producer point of view

Pr 1. The production cost for a record company are \$18 per record and if they produce 60 records, then the total costs are \$1652. The company sells each record for \$40.

(a) Determine the linear cost function, $C(x)$, for the company on production of these records.

(b) Determine the linear revenue function, $R(x)$, for the company's sales of these records.

(c) Determine the linear profit function, $P(x)$, for the company on the production and sale of these records.

Pr 2. Consumers will buy 10,000 items at a price of \$120 per item. If the price goes up by \$30 per item, then they will only buy 7600 items. Producers will not market this item below \$40, but if the price per item increases by \$15, the producers will provide 6000 items to the market.

(a) Determine the linear demand function, $p(x)$.

(b) Determine the linear demand function, $p(x)$.

Pr 3. Given x represents the number of basic tennis rackets supplied or demanded each year, thousands, p represents the price per basic racket, in dollars,

Equation A is $26700x + 329p - 315182 = 0$ and Equation B is $1100x - 47p + 12690 = 0$,

answer the following.

(a) Which equation is the supply equation? Why?

(b) How many basic tennis rackets will consumers purchase if the rackets are free?

(c) Producers will only provide the rackets if the price is above what value?

Section 2.3: Systems of Two Equations in Two Unknowns

- Independent Systems have exactly one solution and $m_1 \neq m_2$
- Inconsistent Systems have no solution and $m_1 = m_2$ AND $b_1 \neq b_2$
- Dependent Systems have infinitely many solutions and $m_1 = m_2$ AND $b_1 = b_2$
- Methods for Solving a System of Two Linear Equations in Two Unknowns
  - Graphical Method
  - Substitution
  - Addition Method
- Parametric Solutions - for dependent systems
- Break-even Points, $(x, R(x))$, where $R(x) = C(x)$ or $P(x) = 0$
- Equilbrium Points, $(x, p(x))$, where $D(x) = S(x)$

Pr 1. State the type of linear system given without graphing or actually computing the solution. Then, state the number of solutions.

(a) $\begin{cases} y = \frac{3}{7}x + \frac{19}{2} \\ y = \frac{3}{7}x - \frac{18}{5} \end{cases}$

(b) $\begin{cases} 4x + 5y = 12 \\ 2x = -\frac{5}{2}y + 6 \end{cases}$

(c) $\begin{cases} -y = -7x + 32 \\ x = -\frac{3}{2}y + \frac{19}{2} \end{cases}$

Pr 2. Find the value of k so that the following system of equations has exactly one solution.

$$\begin{cases} -2x + 3y = 9 \\ kx - 2y = -6 \end{cases}$$

Pr 3. Find the value of k so that the following system of equations has no solution.

$$\begin{cases} y = \frac{5}{4}x + 1 \\ 10x - ky = -6 \end{cases}$$

Pr 4. Given $\frac{2}{5}x - \frac{1}{4}y = \frac{124}{7}$, write an equation such that the sytem of this and your equation would be dependent.

Pr 5. Solve each system using the stated method. Write any solutions as ordered pairs with exact values. For parametric solutions use p as your parameter.

(a) $\begin{cases} 3x + 2y = 5 \\ y = -\frac{3}{2}x + 2 \end{cases}$, using the graphical method.

(b) $\begin{cases} 3x - 2y = -3 \\ 5x - y = 2 \end{cases}$, using the substitution method.

(c) $\begin{cases} 3x - 2y = -4 \\ 4y = 6x + 8 \end{cases}$, using the addition method.

(d) $\begin{cases} x - \frac{3}{2}y = \frac{5}{2} \\ \frac{4}{3}x = -\frac{2}{3}y + 6 \end{cases}$, using the substitution method.

(e) $\begin{cases} 3x + 5y + 2 = 0 \\ -9x - 15y - 6 = 0 \end{cases}$, using the addition method.

(f) $\begin{cases} \frac{x}{9} + \frac{y}{6} = \frac{1}{3} \\ 2x + 3y = 7 \end{cases}$, using the addition method.

Pr 6. The production cost for a record company are \$18 per record and if they produce 60 records, then the total costs are \$1652. The company sells each record for \$40. Determine and interpret the break-even point for the record company on the production and sale of these records.

Pr 7. Nathan operates a geography tutoring stand. His monthly rent for the stand is \$45 and he has to pay A&M \$0.75 for each question that he answers.

(a) What should Nathan charge to answer each question if he wants to make a profit of \$15 when answering 40 questions?

(b) How many questions does he have to answer so that he will break-even?

(c) Interpret the break-even point in the context of the problem.

Pr 8. Consumers will buy 10,000 items at a price of \$120 per item. If the price goes up by \$30 per item, then they will only buy 7600 items. Producers will not market this item below \$40, but if the price per item increases by \$15, the producers will provide 6000 items to the market. Determine and interpret the market equilibrium point for these items.

Pr 9. Given x represents the number of basic tennis rackets supplied or demanded each year, thousands, p represents the price per basic racket, in dollars, Equation A is $267x + 329p - 315182 = 0$ and

Equation B is $11x - 47p + 12690 = 0$, determine the market equilibrium point for these rackets.

Section 2.4: Setting Up and Solving Systems of Linear Equations

- Augmented matrix
- Row Operations
- Reduced Row-Echelon Form
- Gauss-Jordan Elimination Method and pivoting
- rref on the calculator

Pr 1. Set up, but do not solve, a system of linear equations which could be used to solve the problem.

You have $50,000 to invest in Fund A and Fund B. Fund A pays 7.4% and Fund B pays 9.8%. How much do you invest in each to get a return of $4,072 per year?

Pr 2. Set up, but do not solve, a system of linear equations which could be used to solve the problem.

Celia had one hour to spend at the athletic club, where she will jog, play handball, and ride a bicycle. Jogging uses 11 calories per minute, handball 13 and cycling 5. She spends twice as much time jogging than riding a bicycle. How long should she participate in each of these activities in order to use 660 calories?

Pr 3. Write the corresponding augmented matrix for each of the following systems of linear equations.

(a) $\begin{cases} 3x - 2y = -4 \\ 4y = 6x + 8 \end{cases}$

(b) $\begin{cases} 4x + 2y + 3z = 72 \\ 2y - 3z = 12 \\ -x + 9 = 5y + z \end{cases}$

Pr 4. Write a system of linear equations which corresponds to the augmented matrix. Assume the variables are x and y or x, y, and z.

(a) $\left[\begin{array}{cc|c} 2 & -1 & 3 \\ 0 & 3 & 5 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} -2 & -6 & -10 & -12 \\ 0 & 1 & 2 & 3 \\ 2 & 1 & 2 & -5 \end{array}\right]$

Pr 5. Perform the indicated row operation and write the resulting matrix.

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 5 & 2 & 19 \end{array}\right] \xrightarrow{-5R_1 + R_2 \to R_2}$$

Pr 6. Determine the next row operation used in the Gauss-Jordan Elimination method and write the resulting matrix.

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & 5 & 2 & 13 \end{array}\right] \xrightarrow{?}$$

Pr 7. State if the matrix is in reduced row-echelon form. If the matrix is not in reduced row-echelon form, state which of the four conditions is first violated, as stated in the definition.

(a) $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & 1 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 7 & -12 \\ 0 & 1 & -5 & 14 \end{array}\right]$

Pr 8. Solve each system of linear equations using matrices. Write your answer as an ordered pair or ordered triple, as appropriate. For parametric solutions use t as your parameter.

(a) $\begin{cases} 3x + 5y = -2 \\ -9x - 15y = 6 \end{cases}$

(b) $\begin{cases} 3y + x + z = 10 \\ 2x + 7y + 10 = 31 + z \\ z + 4x = 41 - 13y \end{cases}$

(c) $\begin{cases} x + 4z = 0 \\ x + y = -2z + 1 \\ 6z - 3x = -3y + 15 \end{cases}$