Section 3.1: Setting Linear Programming Problems

- Always Define Your Variables
- Objective Function = must include units
- Constraints ≥ ≤

Pr 1. Set up, but do not solve.

An investment company has two funds, A and B, that you can pick from for your personal investments. Each unit of fund A costs $15, yields an annual return of 6%, and has a risk index of 2 per unit. Each unit of fund B costs $12, yields an annual return of 5%, and has a risk index of 1.5 per unit. You have $42,000 available for investing and want to earn at least $2,400 in interest in the coming year. How many units of each fund should you purchase in order to meet your goals and also to minimize the total risk index for your portfolio?

Variables:

\[ a := \text{the number of units of fund A purchased} \]
\[ b := \text{the number of units of fund B purchased} \]
\[ I := \text{the total risk index of the portfolio} \]

Objective: Maximize/Minimize \[ I = 2a + 1.5b \]

Subject to:

\[ 15a + 12b \leq 42000 \] (amount to invest)
\[ 0.06(15a) + 0.05(12b) \geq 2400 \] (annual interest goal)
\[ 0.9a + 0.66b \geq 2400 \] (non-negativity)

\[ a \geq 0, b \geq 0 \]
Pr 2. Set up, but do not solve.

A housing contractor wants to develop a 41 acre tract of land. He has three types of houses: a small 3 bedroom, a large 3 bedroom and a 4 bedroom house. The small three bedroom house requires $60,000 of capital for a profit of $20,000, the large three bedroom house requires $64,000 of capital for a profit of $25,000, and the four bedroom house requires $80,000 of capital for a profit of $24,000. The small three bedroom needs 4000 labor hours, the large three bedroom needs 3000 labor hours, and the 4 bedroom house needs 3900 labor hours. There are currently 250,000 labor hours available. If the small three-bedroom house is on half an acre, the large 3 bedroom house is on 0.75 acres, the four bedroom house is on 1.5 acres and the contractor has 4.5 million in capital, how many of each type should be built to maximize the profit?

Variables:

\[ x := \text{the number of small 3 bedroom homes built} \]
\[ y := \text{the number of large 3 bedroom homes built} \]
\[ z := \text{the number of 4 bedroom homes built} \]
\[ P := \text{the profit earned, in dollars, for selling these houses} \]

Objective: Maximize

\[ P = 20000x + 25000y + 24000z \]

Subject to:

\[ 4000x + 3000y + 3900z \leq 250000 \] (available labor hours)
\[ 0.5x + 0.75y + 1.5z \leq 41 \] (acres available)
\[ 6000x + 6400y + 8000z \leq 4500000 \] (capital available)
\[ x \geq 0, \ y \geq 0, \ z \geq 0 \] (non-negativity)
Pr 3. Set up, but do not solve.

Your umbrella company makes three models: the Sprinkle, the Storm, and the Hurricane. The amounts of cloth (square yards), metal (pounds), and wood (pounds) used in making each model are given in the table.

<table>
<thead>
<tr>
<th></th>
<th>Cloth</th>
<th>Metal</th>
<th>Wood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprinkle</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Storm</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Hurricane</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

The profit for the Storm is $1, for the Hurricane is $2 and for the Sprinkle is $1. Due to certain agreements, the company can make at most 170 Sprinkle umbrellas. If the company has 300 square yards of cloth, 800 pounds of metal, 600 pounds of wood, how many of each type of umbrella should be produced in order to maximize the profit?

Variables:
- \( p \) := the number of Sprinkle umbrellas made & sold
- \( t \) := the number of Storm umbrellas made & sold
- \( h \) := the number of Hurricane umbrellas made & sold
- \( F \) := the profit, in dollars, from the sale of these umbrellas

Objective: Maximize \( F = 1t + 2h + 1p \)

Subject to:
- \( p + 2t + 2h \leq 800 \) \hspace{1cm} (cloth available)
- \( 2p + t + 3h \leq 800 \) \hspace{1cm} (metal available)
- \( p + 3t + 6h \leq 600 \) \hspace{1cm} (wood available)
- \( p \leq 170 \) \hspace{1cm} (agreement)
- \( p \geq 0, \ t \geq 0, \ h \geq 0 \) \hspace{1cm} (non-negativity)
Pr 4. Set up, but do not solve.

A cellphone store sells two types of cellphone, standard and deluxe. The supplier demands that at least 300 phones be sold a month. In order to keep profits up, the number of standard cellphones sold must be at least twice the number of deluxe cellphones. The store spends $150 a week to market each standard phone and $100 a week to market each deluxe phone. How many of each type of cellphone must be sold to minimize weekly marketing costs? What is the minimum weekly marketing cost?

Variables:

\[ x := \text{the number of standard cellphones sold} \]
\[ y := \text{the number of deluxe cellphones sold} \]
\[ C := \text{the weekly marketing cost, in dollars} \]

Objective: Minimize \[ C = 150x + 100y \]

Subject to: \[ x + y \geq 300 \] (require sales)
\[ x \geq 2y \] (ratio)
\[ x \geq 0, y \geq 0 \] (non-negativity)
Pr 5. Set up, but do not solve.

A company produces three types of blankets (full, queen, and king, at its College Station and Galveston factories. Daily production of each factory for each type of blanket is listed below.

<table>
<thead>
<tr>
<th></th>
<th>Full</th>
<th>Queen</th>
<th>King</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Station</td>
<td>200</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>Galveston</td>
<td>200</td>
<td>400</td>
<td>200</td>
</tr>
</tbody>
</table>

To fulfill a particular order the company must produce at least 12000 full size blankets, 16000 queen size blankets, and 18000 king size blankets. The cost of operating the College Station factory is $4500 per day and the cost of operating the Galveston factory is $6000. The College Station factory must operate at least twice as many days as the Galveston factory. The Galveston factory must operate at least 5 days during this production time. How many days should each factory operate to complete the order at a minimum cost, and what is the minimum cost?

Variables:
- \( n \) := the number of days the College Station factory operates
- \( g \) := the number of days the Galveston factory operates
- \( C \) := the total cost, in dollars, of operating these 2 factories

Objective: Minimize \( C = 4500n + 6000g \)

Subject to:
1. \( 200n + 200g \geq 12000 \) (needed full blankets)
2. \( 200n + 400g \geq 16000 \) (needed queen blankets)
3. \( 600n + 200g \geq 18000 \) (needed king blankets)
4. \( n \geq 2g \) (ratio)
5. \( g \geq 5 \) (Galveston restriction)
6. \( n \geq 0 \) (non-negativity)
7. \( n \geq 2g \) (Galveston restriction)
8. \( g \geq 5 \) (non-negativity)
Pr 6. Set up, but do not solve.

You have $12,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than $9,000. If Stock A earn 3% annual interest, and Stock B earn 4% annual interest, how much money should you invest in each to maximize your annual interest?

**Variables:**
- \( a \) := the amount invested in Stock A, in dollars
- \( b \) := the amount invested in Stock B, in dollars
- \( I \) := the total interest earned, in dollars, on all investments

**Objective:** Maximize \( I = 0.03a + 0.04b \)

**Subject to:**
- \( a + b \leq 12000 \) (available $ to invest)
- \( a \geq 2b \) (ratio)
- \( a \leq 9000 \) (restriction on \( a \))
- \( a \geq 0, b \geq 0 \) (non-negativity)
Pr 7. Set up, but do not solve.

An independent taffy company makes three flavors of taffy: strawberry, lemon, and orange. Each strawberry taffy requires 3 minutes to cool and 1 minute to wrap in paper. Each orange taffy requires 5 minutes to cool and 1.5 minutes to wrap in paper. Each lemon taffy requires 4 minutes to cool and 2 minutes to wrap in paper. There are a total of 1.5 hours available for cooling and 0.5 hours available for wrapping. Determine the production of each taffy to maximize profit if the profit on the sale of each orange, lemon, and strawberry taffy is 75 cents, 60 cents, and 50 cents, respectively, and previous sales indicate that they should produce at least three times as many strawberry taffy as lemon taffy. How many of each flavor should the company make to maximize their profits? What is the maximum profit and is any time leftover in cooling or wrapping?

Variables:
- \( W \) := the number of strawberry taffy made and sold
- \( M \) := the number of lemon taffy made and sold
- \( R \) := the number of orange taffy made and sold
- \( P \) := the total profit, in dollars, on the sales of these taffy

Objective: Maximize \( P = 0.75R + 0.6M + 0.5W \) (dollars)

Subject to:
- \( 3W + 5R + 4M \leq 90 \) (available cooling in minutes)
- \( W + 1.5R + 2M \leq 30 \) (available wrapping in minutes)
- \( W \geq 3M \) (ratio for strawberry and lemon)
- \( W \geq 0, R \geq 0, M \geq 0 \) (non-negativity)
Dave operates a book publishing company that prints its books in two different cities. Due to the cost of printing and storing the books, City A can only print up to 15,000 books per month and City B can only print up to 8,000 books per month. These books are shipped to three colleges, C, D, and E, located across the country. The minimum requirement of the colleges are 4000, 10,000, and 8000, respectively. The shipping costs from city A are $1.15 per book to College C, $1.75 per book to College D, and $1.35 per book to College E, while the shipping costs from city B are $1.00, $1.35, and $1.28 per book, respectively. What shipping schedule should the company use so that the shipping cost are kept to a minimum?

Variable:
- \( j \) := the number of books shipped from City A to College C  
- \( k \) := "  
- \( m \) := "  
- \( n \) := "  
- \( p \) := "  
- \( q \) := "  

Objective: Minimize \( C = 1.15j + 1.75k + 1.35m + 1n + 1.35p + 1.28q \)

Subject to:
- \( j + k + m \leq 15000 \) (restrictions on City A)
- \( n + p + q \leq 8000 \) (restrictions on City B)
- \( j + n \geq 4000 \) (restrictions on College C)
- \( k + p \geq 10000 \) (restrictions on College D)
- \( m + q \geq 8000 \) (restrictions on College E)

\( j \geq 0, k \geq 0, m \geq 0, n \geq 0, p \geq 0, q \geq 0 \)