Section 3.2: Graphing Systems of Linear Inequalities in Two Variables

- Solution set to a linear inequality is half of the coordinate plane, while the solution set to a system of linear inequalities is the region of points that satisfy all of the linear inequalities in the system.
- Boundary Line - the corresponding linear equation for a linear inequality
- True Shading vs. Reverse Shading
- Unbounded vs. Bounded solution sets
- Corner Points

Pr 1. Graph the inequality $4x - 7y < 21$, labeling the boundary line and the solution set with $S$.

```
\begin{tikzpicture}
\draw[thin] (-5,-5) grid (15,15);
\draw[->] (-5,0) -- (15,0) node[above] {$x$};
\draw[->] (0,-5) -- (0,15) node[right] {$y$};
\draw (5,-5) -- (5,15);
\draw (-5,5) -- (15,5);
\draw (5,0) -- (-5,0);
\draw (0,5) -- (0,-5);
\end{tikzpicture}
```

Pr 2. Graph the inequality $-5x + 9y \geq 0$, labeling the boundary line and the solution set with $S$.

```
\begin{tikzpicture}
\draw[thin] (-5,-5) grid (15,15);
\draw[->] (-5,0) -- (15,0) node[above] {$x$};
\draw[->] (0,-5) -- (0,15) node[right] {$y$};
\draw (5,-5) -- (5,15);
\draw (-5,5) -- (15,5);
\draw (5,0) -- (-5,0);
\draw (0,5) -- (0,-5);
\end{tikzpicture}
```
Pr 3. Graph the system of linear inequalities, the solution set with S. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

\[
\begin{align*}
  x - 2y &> 8 \\
  x + y &\leq 6 \\
  x &< 4 \\
  x - 2y &> 8 \\
  x + y &\leq 6 \\
  x &< 4
\end{align*}
\]

Boundary Line:

\(x\)-intercept:

\(y\)-intercept:

Test Point:

Corner Points:
Pr 4. Graph the system of linear inequalities, the solution set with S. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

\[
\begin{align*}
8x + 5y &\leq 40 \\
3x + 5y &\geq 30 \\
x - 4y &\geq 0
\end{align*}
\]

Boundary Line:

\[
\begin{align*}
8x + 5y &\leq 40 \\
3x + 5y &\geq 30 \\
x - 4y &\geq 0
\end{align*}
\]

x-intercept:

g-intercept:

Test Point:

Corner Points:
Pr 5. Graph the system of linear inequalities, the solution set with S. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

\[
\begin{align*}
3x + y &\leq 12 \\
6x + 5y &\geq 30 \\
x + 2y &\leq 14 \\
x &\geq 0, \ y &\geq 0
\end{align*}
\]

3x + y \leq 12 \quad 6x + 5y \geq 30 \quad x + 2y \leq 14 \quad x \geq 0 \quad y \geq 0

**Boundary Line:**

**x-intercept:**

**y-intercept:**

**Test Point:**

**Corner Points:**
Pr 6. Use the graph below to write the corresponding system of linear inequalities.

\[
\begin{align*}
y &= 0 \\
x &= 0 \\
x - y &= 3 \\
x + y &= 11 \\
2x + y &= 15
\end{align*}
\]
Section 3.3: Graphical Solution of Linear Programming Problems

- Feasible Region
- Know the parts of the Fundamental Theorem of Linear Programming
- Method of Corners
  - Set up a linear programming problem algebraically.
  - Graph the constraints and determine the feasible region.
  - Identify the exact coordinates of all corner points of the feasible region.
  - Determine whether or not the linear programming problem will have a solution.
  - If a solution will exist, evaluate the objective function at each corner point and determine the optimal point.
- Leftovers

Pr 1. Use the given feasible region to find the maximum and minimum values of the objective function $Z = -7x + 6y$ over the region, if they exist and where they occur.

![Graphical representation of the feasible region with coordinates labeled: A: (6, 1), B: (0, 14), C: (18, 18), D: (11, 5).]
Pr 2. Use the feasible region to determine the maximum and minimum values of the objective function $z = 2x + y$ over the region, if they exist and where they occur.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: (0, -3)</td>
<td></td>
</tr>
<tr>
<td>B: (0, 0)</td>
<td></td>
</tr>
<tr>
<td>C: (0, 11)</td>
<td></td>
</tr>
<tr>
<td>D: (0, 15)</td>
<td></td>
</tr>
<tr>
<td>E: (4, 7)</td>
<td></td>
</tr>
<tr>
<td>F: (7, 4)</td>
<td></td>
</tr>
<tr>
<td>G: (6, 3)</td>
<td></td>
</tr>
<tr>
<td>H: (3, 0)</td>
<td></td>
</tr>
<tr>
<td>I: (7.5, 0)</td>
<td></td>
</tr>
<tr>
<td>J: (11, 0)</td>
<td></td>
</tr>
</tbody>
</table>
Pr 3. Use the Method of Corners to solve the following linear programming problem.

Objective: Maximize \( P = 12x + 8y \)

Subject to: \( 3x + y \leq 12 \)
\[
6x + 5y \geq 30
\]
\[
x + 2y \leq 14
\]
\[
x \geq 0, y \geq 0
\]
Pr 4. Solve the following linear programming problem, using the Method of Corners:

A cellphone store sells two types of cellphone, standard and deluxe. The supplier demands that at least 300 phones be sold a week. In order to keep profits up, the number of standard cellphones sold must be at least twice the number of deluxe cellphones. The store spends $150 a week to market each standard phone and $100 a week to market each deluxe phone. How many of each type of cellphone must be sold to minimize weekly marketing costs? What is the minimum weekly marketing cost?

Boundary Line:

\[ x\text{-intercept:} \]

\[ y\text{-intercept:} \]

Test Point:
Pr 5. Pies Galore specializes in chocolate cream, lemon meringue, and tart cherry pies. Each chocolate cream pie uses 1 pie crust, 1 serving of whipped cream, and 3 servings of sugar. Each lemon meringue pie uses 1 pie crust, 2 servings of whipped cream, and 3 servings of sugar. Each tart cherry pie uses 2 pie crusts, 1 serving of whipped cream, and 1 serving of sugar. Pies Galore has not received a food shipment in a while and only has 50 pie crusts, 100 servings of whipped cream, and 120 servings of sugar on hand. They sell each chocolate cream pie for $15, each lemon meringue pie for $14, and each tart cherry pie for $20. If Pies Galore discovers they will maximize their revenue by making and selling 38 chocolate cream pies, 6 tart cherry pies and no lemon meringue pies, does Pies Galore have any ingredients leftover, when maximizing their revenue?