Section 3.2: Graphing Systems of Linear Inequalities in Two Variables

- Solution set to a linear inequality is half of the coordinate plane, while the solution set to a system of linear inequalities is the region of points that satisfy all of the linear inequalities in the system.
- Boundary Line - the corresponding linear equation for a linear inequality
- True Shading vs. Reverse Shading
- Unbounded vs. Bounded solution sets
- Corner Points

Pr 1. Graph the inequality $4x - 7y < 21$, labeling the boundary line and the solution set with $S$.

Pr 2. Graph the inequality $-5x + 9y \geq 0$, labeling the boundary line and the solution set with $S$. 
Graph the system of linear inequalities, the solution set with $S$. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

\[
\begin{align*}
x - 2y &> 8 \\
x + y &\leq 6 \\
x &< 4
\end{align*}
\]

**Boundary Line:**

- **$x - 2y = 8$**
- **$x + y = 6$**
- **$x = 4$**

**$x$-intercept:** $x = 8$ (8,0)

**$y$-intercept:** $-2y = 8$ (0,-4)

**Test Point:**

- (0,0): $0 - 2(0) \geq 8$ False
- $0 + 0 \leq 6$ True

**Corner Points:**

$A: (4,-2)$

**The solution set is unbounded.**
Pr 4. Graph the system of linear inequalities, the solution set with $S$. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

Graph:

- **Solid boundary lines**
- $8x + 5y \leq 40$
- $3x + 5y \geq 30$
- $x - 4y \geq 0$

**Boundary Line:**
- $8x + 5y = 40$
- $3x + 5y = 30$
- $x - 4y = 0$

**x-intercept:**
- $(5,0)$
- $(10,0)$
- $(0,0)$

**y-intercept:**
- $(0,8)$
- $(0,6)$
- $(0,0)$

**Test Point:**
- $(0,0)$
- $8(0) + 5(0) \geq 40$
- $0 \leq 40$
- True
- $3(0) + 5(0) \geq 30$
- $0 \geq 30$
- False
- Another Point
- Let $x = 8$
- $8 - 4y = 0$
- $-4y = -8$
- $y = 2$
- $(8,2)$

**Reverse Shading:**
- $3x + 5y = 30$
- $x - 4y = 0$
- $9 + 5y = 40$

**Corner Points:**
- No Solution Set
- No Corner Points
Pr 5. Graph the system of linear inequalities, the solution set with $S$. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

$$
\begin{align*}
3x + y &\leq 12 \\
6x + 5y &\geq 30 \\
x + 2y &\leq 14 \\
x &\geq 0, y &\geq 0
\end{align*}
$$

Boundary Line:

$x$-intercept: 

(4,0) 

(5,0) 

(14,0) 

Vertical Line

$y$-intercept:

(0,12) 

(0,0) 

(0,7)

Test Point:

$(0,0)$ 

$3(0)+0 \leq 12$ 

$0 \leq 12$ 

True

$6(0)+5(0) \geq 30$ 

$0 \geq 30$ 

False

$x+2y=14$ 

$x=0$ 

$y=0$

The solution set is bounded.

A: $x=0$ \iff $6x+5y=30$ 

$6(0)+5y=30$ 

$5y=30$ 

$y=6$ 

$(0,6)$

B: $x=0$ \iff $x+2y=14$ 

$(0)+2y=14$ 

$2y=14$ 

$y=7$ 

$(0,7)$

C: $x+2y=14$ \iff $3x+y=12$ 

$\begin{bmatrix} 1 & 2 & 14 \\ 3 & 1 & 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$ 

$(x,y) = (2,6)$

D: $3x+y=12$ \iff $6x+5y=30$ 

$\begin{bmatrix} 3 & 1 & 12 \\ 6 & 5 & 30 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$ 

$(x,y) = \left( \frac{4}{3}, 2 \right)$

Corner Points:

A: $(0,6)$

B: $(0,7)$

C: $(2,6)$

D: $\left( \frac{4}{3}, 2 \right)$
Pr 6. Use the graph below to write the corresponding system of linear inequalities.

Test Point
\[
\begin{array}{c}
10 \\
10 \\
10 -10 \\
10 +10 \\
2(10)+10
\end{array}
\begin{array}{ccc}
\geq & 0 \\
\geq & 0 \\
\leq & 3 \\
\geq & 11 \\
\geq & 15
\end{array}
\]

System:
\[
\begin{cases}
x \geq 0 \\
y \geq 0 \\
x-y \leq 3 \\
x+y \geq 11 \\
x+y \geq 15
\end{cases}
\]
Section 3.3: Graphical Solution of Linear Programming Problems

- Feasible Region
- Know the parts of the Fundamental Theorem of Linear Programming

Method of Corners
- Set up a linear programming problem algebraically.
- Graph the constraints and determine the feasible region.
- Identify the exact coordinates of all corner points of the feasible region.
- Determine whether or not the linear programming problem will have a solution.
- If a solution will exist, evaluate the objective function at each corner point and determine the optimal point.

Leftovers

Pr 1. Use the given feasible region to find the maximum and minimum values of the objective function

\[ Z = -7x + 6y \]

over the region, if they exist and where they occur.

Since the feasible is bounded, a maximum exist and so does a minimum, according to FTLP

The maximum is \( Z = 84 \) at \((x,y) = (0,14)\).
The minimum is \( Z = -47 \) at \((x,y) = (11,5)\).
Pr 2. Use the feasible region to determine the maximum and minimum values of the objective function \( z = 2x + y \) over the region, if they exist and where they occur.

Since the feasible region is unbounded, in \( Q \) \( S \), and the objective function has positive coefficient, then there does not exist a maximum, but there is a minimum.

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(z = 2x + y)</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: (0, -3)</td>
<td>(z = 2(0) + 15)</td>
<td>15 (&gt;) min</td>
</tr>
<tr>
<td>B: (0, 0)</td>
<td>(z = 2(4) + 7)</td>
<td>15</td>
</tr>
<tr>
<td>C: (0, 11)</td>
<td>(z = 2(7) + 4)</td>
<td>18</td>
</tr>
<tr>
<td>D: (0, 15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E: (4, 7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F: (7, 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G: (6, 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H: (3, 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I: (7, 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J: (11, 0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No maximum

*The minimum is \( z = 15 \) at all points on the line segment connecting \((0, 15)\) and \((4, 7)\).*
Pr 3. Use the Method of Corners to solve the following linear programming problem.

Objective: Maximize $P = 12x + 8y$

Subject to: $3x + y \leq 12$

$6x + 5y \geq 30$

$x + 2y \leq 14$

$x \geq 0, y \geq 0$

The solution set is bounded, thus by the FTLP, the objective function has a maximum.

<table>
<thead>
<tr>
<th>Corner Points</th>
<th>$P = 12x + 8y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: (0,0)</td>
<td>$P = 12(0) + 8(0) = 0$</td>
</tr>
<tr>
<td>B: (0,7)</td>
<td>$P = 12(0) + 8(7) = 56$</td>
</tr>
<tr>
<td>C: (2,0)</td>
<td>$P = 12(2) + 8(0) = 24$</td>
</tr>
<tr>
<td>D: ($\frac{10}{3}, 2$)</td>
<td>$P = 12\left(\frac{10}{3}\right) + 8(2) = 56$</td>
</tr>
</tbody>
</table>

The maximum is $P = 72$ when $(x, y) = (2, 6)$. 

See graph and corner pt work on problem 5 of 3.2
Pr 4. Solve the following linear programming problem, using the Method of Corners:

A cellphone store sells two types of cellphone, standard and deluxe. The supplier demands that at least 300 phones be sold a week. In order to keep profits up, the number of standard cellphones sold must be at least twice the number of deluxe cellphones. The store spends $150 a week to market each standard phone and $100 a week to market each deluxe phone. How many of each type of cellphone must be sold to minimize weekly marketing costs? What is the minimum weekly marketing cost?

Objective: Minimize \( C = 150s + 100d \)

Subject to:
1. \( s + d \geq 300 \)
2. \( s \geq 2d \)
3. \( s \geq 0, \ d \geq 0 \)

Boundary Line:
- \( s \)-intercept: \((300, 0)\)
- \( d \)-intercept: \((0, 300)\)

Test Point:
- \((0, 0)\): False

The feasible region is unbounded, QD, and the objective function has positive coefficients, thus the minimum exists.

At \((200, 100)\):
- \( C = 150(200) + 100(100) = 40000 \)
- \( s = 200 \)
- \( d = 100 \)

At \((300, 0)\):
- \( C = 150(300) + 100(0) = 45000 \)
- \( s = 300 \)
- \( d = 0 \)

The company should sell 200 standard and 100 deluxe cellphones in order to minimize their weekly marketing cost at $40,000.
Pr 5. Pies Galore specializes in chocolate cream, lemon meringue, and tart cherry pies. Each chocolate cream pie uses 1 pie crust, 1 serving of whipped cream, and 3 servings of sugar. Each lemon meringue pie uses 1 pie crust, 2 servings of whipped cream, and 3 servings of sugar. Each tart cherry pie uses 2 pie crusts, 1 serving of whipped cream, and 1 serving of sugar. Pies Galore has not received a food shipment in a while and only has 50 pie crusts, 100 servings of whipped cream, and 120 servings of sugar on hand. They sell each chocolate cream pie for $15, each lemon meringue pie for $14, and each tart cherry pie for $20. If Pies Galore discovers they will maximize their revenue by making and selling 38 chocolate cream pies, 6 tart cherry pies and no lemon meringue pies, does Pies Galore have any ingredients leftover, when maximizing their revenue?

**Optimal Solution:** \((38, 6, 0)\)

**Resources**

<table>
<thead>
<tr>
<th></th>
<th>Used</th>
<th>Available</th>
<th>Leftover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pie Crust: 1c + 1m + 2t</td>
<td>((38, 0, 0))</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Whipped Cream: 1c + 2m + 1t</td>
<td>((38, 2, 0))</td>
<td>100</td>
<td>56</td>
</tr>
<tr>
<td>Sugar: 3c + 3m + 1t</td>
<td>((38, 5, 0))</td>
<td>120</td>
<td>0</td>
</tr>
</tbody>
</table>

There were 0 pie crusts, 56 servings of whipped cream, and 0 servings of sugar leftover.