SECTION 4.1: MATHEMATICAL EXPERIMENTS

- Sample space, S - a list of all possible outcomes in the mathematical experiments
- Event - a subset of the sample space
  - Simple Event
  - Certain Event
  - Impossible Event
- Using tree diagrams to determine a sample space in a two-stage experiment
- Venn Diagrams
- Operations on Events
  - Complement, \( A^C \)
  - Intersection, \( A \cap B \)
  - Union, \( A \cup B \)
- Mutually Exclusive Events

Pr 1. State the sample space for each experiment:
   (a) Selecting a letter at random from the word “skate” and noting the letter.

   (b) Identical ping pong balls are numbered 0 to 10, one ping pong ball is drawn at random, noting the number on the ball.

   (c) A standard 30-sided die is rolled and it is noted whether the number is a multiple of 4 or is not a multiple of 4.

   (d) The numbers 0, 1, 2, 3, and 4 are written on separate pieces of paper and put in a hat. Two pieces of paper are drawn at the same time and the product of the numbers is noted.

   (e) A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.
Pr 2. Consider the experiment of selecting a letter at random from the word “skate” and noting the letter.
(a) State all the simple events for the experiment.

(b) State the certain event for the experiment.

(c) Given an example of an impossible event for the experiment.

(d) State the total number of possible events.

(e) Write the outcomes in the event, $J$, “a consonant is draw.”

Pr 3. A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.
(a) State all the simple events for the experiment.

(b) State the certain event for the experiment.

(c) Given an example of an impossible event for the experiment.

(d) State the total number of possible events.

(e) Write the outcomes in the event, $M$, “a black or a number less an 3 is rolled.”
**Pr 4.** Let $A$ and $B$ be two events of the sample space, $S$.

Use a two-circle Venn diagram to illustrate which region(s) contain the outcomes of the resulting events.

**a.** $B^C \cap A$

![Venn diagram illustrating $B^C \cap A$]

**b.** $(A \cup B) \cap A^C$

![Venn diagram illustrating $(A \cup B) \cap A^C$]

**c.** $(A^C \cup B)^C$

![Venn diagram illustrating $(A^C \cup B)^C$]
Pr 5. An experiment consists of rolling a five-sided, noting the number showing uppermost and then spinning a spinner with five equal regions (red, blue, purple, maroon, and green), noting the color.

Let

$V :=$ the event “a number greater than 3 is rolled”

$W :=$ the event “an even is rolled”

$X :=$ the event “the spinner lands on blue”

$Y :=$ the event “the spinner lands on a color other than green”

$Z :=$ the event “the spinner lands on purple or maroon.”

(a) Write the symbolic notation for the event, $D$, that “an odd is rolled or the spinner lands on purple or maroon.”

(b) Write the symbolic notation for the event, $H$, that “a number less than or equal to 3 is rolled or the spinner lands on a color other than green, but not blue.”

(c) Describe the event $X^C \cap W$.

(d) Describe the event $Z \cup Y \cup Y^C$

(e) Are event $V$ and event $W$ mutually exclusive? Explain why or why not.
Section 4.2: Basics of Probability

- Equally Likely Outcomes
- Uniform Sample Space
- Probability of an event = \( \frac{\text{the total number of outcomes in A}}{\text{the total number of possible outcomes}} \)
- Theoretical Probability
- Empirical Probability
- Probability Distributions

Pr 1. Determine if the sample space for each experiment is uniform or not.
(a) Selecting a letter at random from the word “skate” and noting the letter.

(b) Identical ping pong balls are numbered 0 to 10, one ping pong ball is drawn at random, noting the number on the ball.

(c) A standard 30-sided die is rolled and it is noted whether the number is a multiple of 4 or is not a multiple of 4.

(d) The numbers 0, 1, 2, 3, and 4 are written on separate pieces of paper and put in a hat. Two pieces of paper are drawn at the same time and the product of the numbers is noted.

(e) A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.

Pr 2. A card is selected from a well-shuffled standard 52-card deck. Compute each of the following probabilities.
(a) \( P(\text{a 5 is drawn}) \)

(b) \( P(\text{a red card is drawn}) \)

(c) \( P(\text{a black Ace is drawn}) \)

(d) \( P(\text{a card which is not a diamond is drawn}) \)

(e) \( P(\text{a face card is drawn or spade is drawn}) \)
Pr 3. An experiment consists of rolling a five-sided, noting the number showing uppermost and then spinning a spinner with five equal regions (red, blue, purple, maroon, and green), noting the color. What is the probability that

(a) A 3 is rolled

(b) The spinner lands on blue or red.

(c) The spinner lands on a color other than maroon.

(d) An odd number is rolled or the spinner lands on green.

(e) A number greater than 2 and the spinner does not land on purple.
Pr 4. A music store selected 1000 customers at random and surveyed them to determine a relationship between age of purchaser and monthly purchases of CDs. The results are given in the table below.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4 or More</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 12 (A)</td>
<td>50</td>
<td>60</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>170</td>
</tr>
<tr>
<td>12 - 18 (B)</td>
<td>30</td>
<td>100</td>
<td>90</td>
<td>30</td>
<td>40</td>
<td>290</td>
</tr>
<tr>
<td>19 - 25 (C)</td>
<td>70</td>
<td>110</td>
<td>100</td>
<td>30</td>
<td>20</td>
<td>330</td>
</tr>
<tr>
<td>Over 25 (D)</td>
<td>100</td>
<td>50</td>
<td>40</td>
<td>10</td>
<td>10</td>
<td>210</td>
</tr>
<tr>
<td>Totals</td>
<td>250</td>
<td>320</td>
<td>260</td>
<td>90</td>
<td>80</td>
<td>1000</td>
</tr>
</tbody>
</table>

If a surveyed person is selected at random, compute each of the following.

(a) $P(B)$

(b) $P(C^C)$

(c) $P(C \cap 3)$

(d) $P(A \cup 1)$

(e) $P((B \cup D)^C \cap 4)$

Pr 5. Is the following probability distribution valid? If valid, does the distribution represent an experiment with uniform sample space?

<table>
<thead>
<tr>
<th>Outcome</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$\frac{2}{25}$</td>
<td>$\frac{1}{25}$</td>
<td>$\frac{10}{25}$</td>
<td>$\frac{8}{25}$</td>
<td>$\frac{4}{25}$</td>
</tr>
</tbody>
</table>