Math 140 - Spring 2021
Week In Review §7 - Mar. 15, 2021

Section 4.3: Rules of Probability

- The probability of an event, \( A \), is always between 0 and 1, inclusively. \( 0 \leq P(A) \leq 1 \)
- The probability of an impossible event is 0. \( P(\emptyset) = 0 \)
- The probability of the certain event is 1. \( P(S) = 1 \)
- Union Rule: \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
- Complement Rule: \( P(A^C) = 1 - P(A) \)

Pr 1. Let \( S = \{s_1, s_2, s_3, s_4\} \) be the sample space for an experiment with the distribution given below.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{3}{25} )</td>
<td>( \frac{4}{25} )</td>
<td>( \frac{18}{50} )</td>
<td></td>
</tr>
</tbody>
</table>

Let \( A = \{s_1, s_3\} \) and \( B = \{s_1, s_4\} \).

(a) Fill in the missing probability in the distribution table.

Determine the following probabilities.

(b) \( P(A) \)

(c) \( P(A \cup B) \)

(d) \( P(A \cap B) \)

(e) \( P((A \cap B)^C) \)

(f) \( P(A^C \cup B) \)
Pr 2. A fair standard five-sided die is rolled, noting the number shown, and a fair coin was flipped, noting the side showing. What is the probability that
(a) An even number is rolled or a tails was showing.

(b) A 3 is rolled and a tails is showing.

(c) The coin does not show a heads.

Pr 3. Let $A$ and $B$ be two events of an experiment. Suppose $P(A) = 0.65, P(B) = 0.62,$ and $P(A \cup B) = 0.84.$ Calculate the following probabilities:
(a) $P(A^C)$

(b) $P(A \cap B)$

(c) $P(A^C \cup B^C)$

(d) $P((A \cup B)^C)$
Section 4.4: Probability Distributions

- Random Variables
- Expected Value $E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$
- Premiums
- Fair Games $E(\text{net winnings}) = 0$

Pr 1. The probability distribution for tossing a coin three times and counting the number of tails is given below.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

(a) Compute the probability that more than one head is tossed.

(b) Compute the probability that four heads are tossed.

(c) State the expected number of heads in an experiment where three coins are tossed.
Pr 2. You are going on a European vacation and decide to purchase travel insurance on your brand new luggage worth $1500. The insurance policy will cost $48. In the event your luggage is damaged to the point of needing duct tape, then you will receive 50% of the value of the luggage. In the event your luggage is lost or stolen, then you will receive 100% of the value of the luggage. According to airline data, the probability of your luggage being damaged and needing duct tape is 1%, while the probability your luggage is lost or stolen is 0.8%. Let $X$ be the insurance company’s net gain or loss on the policy described.

(a) Create a probability distribution for $X$.

(b) Compute the insurance company’s expected profit for this policy.

Pr 3. You purchase a brand new yacht for $1,000,000 and insure it. The policy pays 40% of the yacht’s value if it is involved in a minor accident (as defined by the insurance company) or 90% of the yacht’s value if it sinks. The probability of a minor yachting accident is 0.25, while the probability your yacht sinks is 0.005. What is the minimum amount the insurance company will charge for this policy.
Pr 4. A real estate investor buys a parcel of land for $225,000. He estimates the probability that he can sell it for $375,000 to be 24%, the probability that he can sell it for $200,000 to be 35%, and the probability that he can sell it for $189,000 to be 41%. What is the expected profit from the sale of this land?

Pr 5. You play a game where a card is drawn from a well-shuffled standard deck of 52 cards, noting the color of the card, and a spinner divided into four equal regions (red, blue, green, and yellow) is spun, noting the color. If the spinner lands on a color other than yellow, you win $3. If the color of the card is red and the spinner lands on red, you win $10. Otherwise you lose. Let $X$ be your winnings.

a. Create a probability distribution for $X$.

b. Compute your expected winnings for the game.

c. How much should be charged in order to make the game fair?