Note #1 (Vectors, Dot Product.)

[Vectors]

1. Find a vector $\vec{AB}$.
   
   a. $A(-3, 4), B(1, -2)$
   
   b. $A(0, 0), B(1, 1)$
   
   c. $A(-2, 2), B(-1, 3)$
(2) If \( \mathbf{a} = \langle -1, 2 \rangle \) and \( \mathbf{b} = \langle 5, 3 \rangle \), find \( |\mathbf{a}| \), \( \mathbf{a} + \mathbf{b} \), \( \mathbf{a} - \mathbf{b} \), and \( -3\mathbf{a} + 4\mathbf{b} \).

(3) If \( |\mathbf{r}| = 2 \), and \( \mathbf{r} \) makes an angle of 210° with the positive \( x \)-axis, find the component of the vector \( \mathbf{r} \).

(4) If \( \mathbf{a} = \langle 3, -4 \rangle \), find a vector with length 10 in the direction of \( \mathbf{a} \).
[Dot product]

(5) Find \( \mathbf{a} \cdot \mathbf{b} \).

(a) \(|\mathbf{a}| = 4, \, |\mathbf{b}| = 5\), the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \frac{\pi}{3} \)

(b) \( \mathbf{a} = \langle -2, -8 \rangle, \, \mathbf{b} = \langle 6, -4 \rangle \)

(c) \( \mathbf{a} = \mathbf{i} + \mathbf{j}, \, \mathbf{b} = \mathbf{i} - 2\mathbf{j} \)
(6) Find the angle between the vectors.
(a) \( \mathbf{a} = \langle 6, 0 \rangle , \mathbf{b} = \langle 5, 3 \rangle \)

(b) \( \mathbf{a} = \langle 3, 1 \rangle , \mathbf{b} = \langle 2, 4 \rangle \)
(7) Find the values of $x$ such that the given vectors are orthogonal.

(a) $\langle 4, x \rangle, \langle x, 1 \rangle$

(b) $\langle x, x \rangle, \langle 1, x \rangle$
(8) A force $\mathbf{F} = \langle -3, 4 \rangle$ is used to move an object from the point $(0, 2)$ to the point $(-3, 3)$. How much work is done by the force if distance is measured in meters and force is measured in Newtons?
(9) A boat heads in the direction $N30^\circ E$ with a speed of $40mph$. The water current is flowing $S45^\circ E$ with a speed of $6mph$. Find the true speed and direction of the boat.