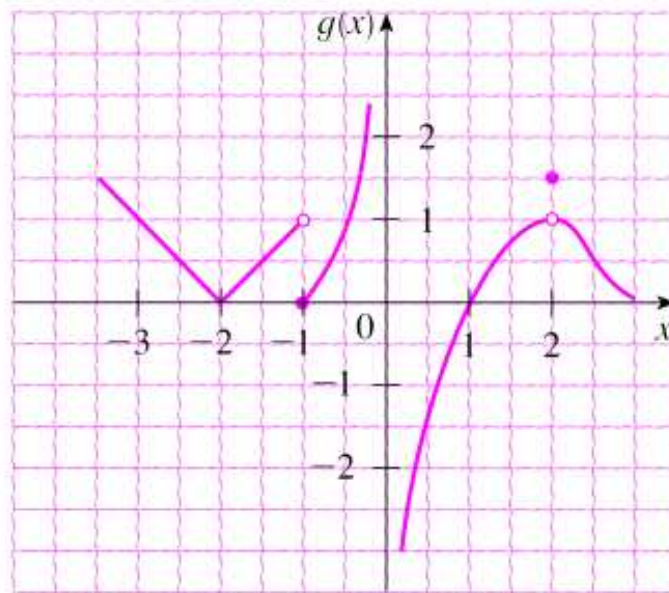




NOTE #3 (THE LIMIT OF A FUNCTION, CALCULATING LIMITS USING THE
LIMIT LAWS, CONTINUITY)

[The Limit of a Function]

(1) State the value of the given quantity, if it exists, from the given graph.



a. $\lim_{x \rightarrow 0^-} g(x)$

b. $\lim_{x \rightarrow 0^+} g(x)$

c. $\lim_{x \rightarrow 0} g(x)$

d. $\lim_{x \rightarrow 2^-} g(x)$

e. $\lim_{x \rightarrow 2^+} g(x)$

f. $\lim_{x \rightarrow 2} g(x)$

g. $g(2)$

h. $\lim_{x \rightarrow -1^-} g(x)$

i. $\lim_{x \rightarrow -1^+} g(x)$

j. $\lim_{x \rightarrow -1} g(x)$



(2) Find the limit.

(a) $\lim_{x \rightarrow 3} \frac{1}{(x-3)^8}$

(b) $\lim_{x \rightarrow 0} \frac{x-1}{x^2(x+2)}$

(c) $\lim_{x \rightarrow -2^+} \frac{x-1}{x^2(x+2)}$

(d) $\lim_{x \rightarrow -2^-} \frac{x-1}{x^2(x+2)}$

(e) $\lim_{x \rightarrow -2} \frac{x-1}{x^2(x+2)}$



[Calculating Limits Using the Limit Laws]

(3) Evaluate the limit.

(a) $\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$

(b) $\lim_{x \rightarrow 1} \frac{x^2 - x - 2}{x + 1}$

(c) $\lim_{t \rightarrow 0} \frac{\sqrt{2-t} - \sqrt{2}}{t}$



$$(d) \lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$$

$$(e) \lim_{t \rightarrow 1} \left\langle 2t - 3, \frac{t^2 - t}{t - 1} \right\rangle$$



(4) Find the limit.

(a) $\lim_{x \rightarrow -4^-} \frac{|x + 4|}{x + 4}$

(b) $\lim_{x \rightarrow -4^+} \frac{|x + 4|}{x + 4}$

(c) $\lim_{x \rightarrow -4} \frac{|x + 4|}{x + 4}$



(5) Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

evaluate each of the following limits if it exists.

(a) $\lim_{x \rightarrow 0^+} f(x)$

(b) $\lim_{x \rightarrow 0^-} f(x)$

(c) $\lim_{x \rightarrow 0} f(x)$

(d) $\lim_{x \rightarrow 1} f(x)$

(e) $\lim_{x \rightarrow 2^-} f(x)$

(f) $\lim_{x \rightarrow 2^+} f(x)$

(g) $\lim_{x \rightarrow 2} f(x)$



[Continuity]

(6) Explain why the function $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ is discontinuous at $x = 0$.



(7) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$