Note ♯4 (Exam1 Review)

(1) Find the scalar and vector projections of $\langle -3, 1\rangle$ onto $\langle 2, 5\rangle$.

key: scalar projection: $-\dfrac{1}{\sqrt{29}}$, *vector projection:* $\left\langle -\dfrac{2}{29}, -\dfrac{5}{29} \right\rangle$

(2) Vector **a** starts at the point $(-3, 1)$ and ends at the point $(6, 3)$. Find a unit vector of **a**.

key: $\left\langle \dfrac{9}{\sqrt{85}}, \dfrac{2}{\sqrt{85}} \right\rangle$

(3) Determine a vector equation of the straight line which passes through the point $(1, -1)$ and is parallel to the vector $\langle -2, 3\rangle$.

key: $\langle 1 - 2t, -1 + 3t\rangle$

(4) Find the angle $\angle ACB$ of the triangle with the vertices, $A(3,0)$, $B(0,3)$, $C(5,4)$.

key: $\theta = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$

(5) Find the distance from the point $(1,2)$ to the line $y = 3x - 4$.

key: $\frac{3}{\sqrt{10}}$

(6) Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on an object. The magnitude of $\mathbf{F_1}$ is 10 lb and it makes a $120°$ angle with the positive x-axis. The magnitude of $\mathbf{F_2}$ is 8 lb and it makes a $45°$ angle with the positive x-axis. Find the magnitude of the resultant force $\mathbf{F}$.

(7) A constant force with the vector representation $\mathbf{F} = \langle 10, 18 \rangle$ moves an object along a straight line from the point $(2, 3)$ to the point $(4, 9)$. Find the work done, if the distance is measured in meters and the magnitude of the force is measured in newtons.

key: $128\ J$

(8) A woman exerts a horizontal force of 65 lb on a crate as she pushes it up a ramp that is 20 ft long and inclined at an angle of $30°$ above the horizontal. Find the work done on the box.

key: $650\sqrt{3}\ ft \cdot lb$

(9) Find the equation of the line perpendicular to the vector $\langle 3, 5\rangle$ and passing through the point $(5, 1)$.

key: $\mathbf{r(t)} = \langle 5 - 5t, 1 + 3t\rangle$

(10) Consider the line $x = 8 - 2t$, $y = 14 + 7t$. (a) Find a vector perpendicular to the line, (b) Find the Cartesian form and (c) sketch the graph.

key: vector perpendicular: $\langle -7, -2\rangle$, $\langle 7, 2\rangle$, *Cartesian form:* $y = -\dfrac{7}{2}x + 42$

(11) Simplify.

key

(a) $\tan(\arccos(\frac{1}{4}))$

(b) $\sin(\arctan(2))$

(c) $\tan(\arcsin(3x))$

(12) Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

evaluate each of the following limits if it exists.

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to 2^+} f(x)$

(f) $\lim_{x \to 2} f(x)$

(13) Evaluate the limit.

key

(a) $\lim_{x\to 2} \frac{x-1}{x^2+4}$

(b) $\lim_{x\to -1} \frac{x^2-x-2}{x+1}$

(c) $\lim_{t\to 0} \frac{\sqrt{2-t}-\sqrt{2}}{t}$

(d) $\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$

(e) $\lim_{x \to -4^-} \frac{|x+4|}{x+4}$

(f) $\lim_{x \to 4^-} \frac{|x-4|}{x^2 - 2x - 8}$

(g) $\lim_{x \to -1^-} \frac{x-2}{x+1}$

(h) $\lim_{x \to -\infty} \frac{-x^3+2x^2-4x}{8+4x^2-5x^3}$

(i) $\lim_{x \to \infty} \frac{e^x+2e^{-x}}{2e^x-e^{-x}}$

(j) $\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{5-2x^3}$

(k) $\lim_{x\to\infty} \ln\left(\frac{x}{x^2-3x}\right)$

(l) $\lim_{x\to\infty} \arctan(e^x)$

(m) $\lim_{x\to\infty} \arctan(\ln x)$

(n) $\lim_{x\to 0^+} \arctan(\ln x)$

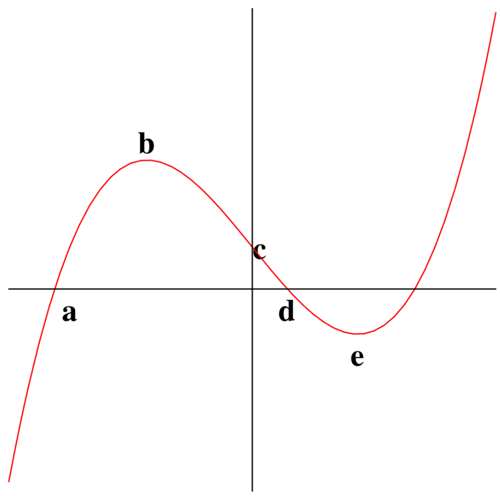
(14) Given $-2x - 2 \leq f(x) \leq \frac{1}{2}x^2$, compute $\lim_{x \to -2} f(x)$.

key: 2

(15) Find the vertical and horizontal asymptotes of $f(x) = \frac{(x-1)(x+3)}{x^2-1}$.

key: V.A: $x = -1$, *H.A:* $y = 1$

(16) Given the graph of $f(x)$ below, sketch the graph of the derivative.



key: See the video

(17) Given $f(x) = \begin{cases} x - 4a & \text{if } x < -2 \\ ax^2 & \text{if } x \geq -2 \end{cases}$. Find the value of a which makes the function continuous everywhere.

key: $a = -\frac{1}{4}$

(18) Which of following intervals must contain a solution to the equation $2x^3 + 16x + 3 = 22$?

(a) $[-2, -1]$
(b) $[-1, 0]$
(c) $[0, 1]$
(d) $[1, 2]$
(e) $[2, 3]$

key: d

(19) Find the average rate of change of $f(x) = x^2 + 6$ from $x = -3$ to $x = 1$.

key: -2

(20) Fine $f'(x)$ using the limit definition of the derivative for $f(x) = 3x^2 - 4$.

key: $f'(x) = 6x$

(21) Using the result above, find the equation of the tangent line to the graph of $f(x) = 3x^2 - 4$ at $x = 2$.

(22) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \sqrt{3x+1}$.

key: $f'(x) = \dfrac{3}{2\sqrt{3x+1}}$

(23) Using the result above, find the equation of the tangent line to the graph of $f(x) = \sqrt{3x+1}$ at $x = 1$.

(24) Find $f'(x)$ using the limit definition of the derivative for $f(x) = \frac{-2}{x+2}$.

key: $f'(x) = \frac{2}{(x+2)^2}$

(25) Using the result above, find the equation of the tangent line to the graph of $f(x) = \frac{-2}{x+2}$ at $x = 0$.