Note §6 (Derivatives of Logarithmic Functions, Derivatives of Vector Functions, Slopes and Tangents to Parametric Curves, Rates of Change in the Natural and Social Sciences)

[Derivatives of Inverse Trigonometric Functions and Logarithmic Functions]

(1) Find the derivative of the function.
   (a) $y = (\tan^{-1} x)^2$

   (b) $y = \tan^{-1}(x^2)$
(c) \( R(t) = \arcsin(1/t) \)

(d) \( f(x) = \ln(\sin^2 x) \)

(e) \( g(x) = \ln(xe^{-2x}) \)
(f) \( f(x) = \log(1 + \cos x) \)

(g) \( F(s) = \ln(\ln s) \)

(h) \( y = \log_2(x \log_5 x) \)
(2) If \( f(x) = \cos(\ln x^2) \), find \( f'(1) \).

(3) Find an equation of tangent line to the curve \( y = x^2 \ln x \) at the point \((1, 0)\).
[Logarithmic Differentiation]

(4) Use the logarithmic differentiation to find the derivative of the function.

(a) \( y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1} \)
(b) \( y = x^x \)
(c) \( y = (\ln x)^\cos x \)
[Derivatives of Vector Functions]

(5) Sketch the curve with the given vector equation. Indicate with an arrow the direction in which \( t \) increases.

(a) \( r(t) = \langle 2t, t^3 + 1 \rangle \)
(b) \( r(t) = \langle \sin t, \cos t \rangle \)
[Slopes and Tangents to Parametric Curves]

(6) Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

(a) \( x = 1 - t^3, \ y = t^2 - 3t + 1; \ t = 1 \)
(b) \( x = 2 \sin \theta, \ y = 3 \cos \theta; \ \theta = \pi/4 \)
(7) Find the point(s) on the curve \( x = t(t^2 - 3), \ y = 3(t^2 - 3) \) where the tangent is horizontal or vertical.
(8) A particle moves according to the equation of motion $s(t) = t^3 - 9t^2 + 15$, where $s(t)$ is measured in meters and $t$ in seconds.

(a) Find the velocity at time $t$.

(b) When is the particle at rest?

(c) The acceleration at the times when the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the distance traveled in the first 6 seconds.