Note 9 (Optimization Problems, Antiderivatives)

[Optimization Problems]
(1) If 10,800 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
(2) A company wants to manufacture a box with a volume of 36 cubic feet. The box has no top, and the length is twice of width. Find the dimensions of the box that minimizes the amount of material used.
(3) The top and bottom margins of a poster are each 9 cm and the side margins are each 6 cm. The area of printed material on the poster is fixed at 864 cm². Find the dimensions of the printed area that minimize the area of the whole poster.
(4) Find the dimensions of the rectangle of largest area that has its base on the $x$-axis and its other two vertices above the $x$-axis and lying on the parabola $y = 6 - x^2$. 
(5) Find the point on the line $5x + y = 7$ that is closest to the point $(-2, 1)$. 
(6) A rectangular storage container with an open top is to have a volume of 10 $m^3$. The length of this base is twice the width. Material for the base costs $5 per square meter. Material for the sides costs $3 per square meter. Find the cost of materials for the cheapest such container.
(7) Find the most general antiderivative of the function.

(a) \( f(x) = 6x^5 - 8x^4 - 9x^2 \)

(b) \( f(x) = (x - 5)^2 \)
(c) \( g(t) = \frac{1 + t + t^2}{\sqrt{t}} \)

(d) \( g(x) = 2 \cos x - \frac{3}{\sqrt{1 - x^2}} \)
(8) Find $f$.
   (a) $f''(x) = 2x^3 - 12x^2 + 6x$

   (b) $f''(x) = 1/x^2$
(c) $f'(x) = 5x^4 - 3x^2 + 4, \quad f(-1) = 2$

(d) $f''(\theta) = \sin \theta + \cos \theta, \quad f(0) = 3, \quad f'(0) = 4$
(e) \( f''(x) = 4 + 6x + 24x^2, \quad f(0) = 3, \quad f(1) = 10 \)
(9) A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?