Wir 5: Sections 14.5, 14.6

Section 14.5

Problem 1. If \( z = \ln(9x - 6y) \), \( x = \cos(e^t) \), \( y = \sin^3(4t) \), find \( \frac{dz}{dt} \).

Problem 2. If \( w = u^2 + 2uv \), \( u = r \ln s \), \( v = 2r + s \), find \( \frac{\partial w}{\partial r} \) and \( \frac{\partial w}{\partial s} \).

Problem 3. If \( z = x^4 + xy^3 \), \( x = uv^3 + w^4 \), \( y = u + ve^w \), find \( \frac{\partial z}{\partial u} \) when \( u = 1 \), \( v = 1 \), \( w = 0 \).

Problem 4. The height and radius of a right circular cone are changing with respect to time. If the base radius of the cone is increasing at a rate of \( \frac{1}{4} \) inches per minute while the height is decreasing at a rate \( \frac{1}{10} \) inches per minute, find the rate in which the volume of the cone is changing when the radius of the cone is 2 inches and the height of the cone is 1 inch.

Problem 5. The length \( l \), width \( w \) and height \( h \) of a box change with time. At a certain instant, the dimensions are \( l = 1 \) m, \( w = 3 \) m and \( h = 2 \) m, and \( l \) and \( w \) are increasing at rate of 2 m/s while \( h \) is decreasing at a rate of 3 m/s. At that same instant, find the rate at which the surface area is changing.

Section 14.6

Problem 6. \( f(x, y) = xy \sin x \), find the directional derivative at the point \( \left( \frac{\pi}{2}, -1 \right) \) in the direction \( u = \left\langle 3, 4 \right\rangle \).

Problem 7. Given \( f(x, y) = x^3y^2 \), find the directional derivative at the point \( (-1, 2) \) in the direction \( 4i - 3j \).

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.
Problem 8. If \( f(x, y) = x^2 e^{xy} \), find the rate of change of \( f \) at the point \((1, 0)\) in the direction of the point \( P(1, 0) \) to the point \( Q(5, 2) \).

Problem 9. Find the gradient of \( f(x, y) = x^2 + y^2 - 4xy \) at the point \((1, -1)\).

Problem 10. If \( f(x, y) = x^2 e^{-2y} \), \( P(2, 0) \), \( Q(-3, 1) \).
   a.) Find the directional derivative at \( Q \) in the direction of \( P \).
   b.) Find a vector in the direction in which \( f \) increases most rapidly at \( P \), and find the rate of change of \( f \) in that direction.

Problem 11. Find the maximum rate of change of \( f(x, y) = \sin^2(3x + 2y) \) at the point \( \left( \frac{\pi}{6}, -\frac{\pi}{8} \right) \) and the direction in which it occurs.

Problem 12. Find the equation of the tangent plane to the surface \( f(x, y) = x^2 + y^2 - 4xy \) at the point \((1, 2)\).