TEXAS A&M UNIVERSITY
Math Learning Center

Math 251 – Spring 2021
Week In Review

Wir 5: Sections 14.5, 14.6

Section 14.5

Problem 1. If $z = \ln(9x - 6y)$, $x = \cos(e^t)$, $y = \sin^3(4t)$, find $\dfrac{dz}{dt}$.

Problem 2. If $w = u^2 + 2uv$, $u = r \ln s$, $v = 2r + s$, find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$.

Problem 3. If $z = x^4 + xy^3$, $x = uv^3 + w^4$, $y = u + ve^w$, find find $\dfrac{\partial z}{\partial u}$ when $u = 1$, $v = 1$, $w = 0$.

Problem 4. The height and radius of a right circular cone are changing with respect to time. If the base radius of the cone is increasing at a rate of $\dfrac{1}{4}$ inches per minute while the height is decreasing at a rate $\dfrac{1}{10}$ inches per minute, find the rate in which the volume if the cone is changing when the radius of the cone is 2 inches and the height of the cone is 1 inch.

Problem 5. The length l, width w and height h of a box change with time. At a certain instant, the dimensions are $l = 1$ m, $w = 3$ m and $h = 2$ m, and l and w are increasing at rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that same instant, find the rate at which the surface area is changing.

Section 14.6

Problem 6. $f(x, y) = xy \sin x$, find the directional derivative at the point $\left(\frac{\pi}{2}, -1\right)$ in the direction $\mathbf{u} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$.

Problem 7. Given $f(x, y) = x^3y^2$, find the directional derivative at the point $(-1, 2)$ in the direction $4\mathbf{i} - 3\mathbf{j}$.

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.

Problem 8. If $f(x, y) = x^2 e^{xy}$, find the rate of change of f at the point $(1, 0)$ in the direction of the point $P(1, 0)$ to the point $Q\,(5, 2)$.

Problem 9. Find the gradient of $f(x, y) = x^2 + y^3 - 4xy$ at the point $(1, -1)$.

Problem 10. If $f(x, y) = x^2 e^{-2y}$, $P(2, 0)$, $Q(-3, 1)$.

a.) Find the directional derivative at Q in the direction of P.

b.) Find a vector in the direction in which f increases most rapidly at P, and find the rate of change of f in that direction.

Problem 11. Find the maximum rate of change of $f(x, y) = \sin^2(3x + 2y)$ at the point $\left(\frac{\pi}{6}, -\frac{\pi}{8}\right)$ and the direction in which it occurs.

Problem 12. Find the equation of the tangent plane to the surface $f(x, y) = x^2 + y^2 - 4xy$ at the point $(1, 2)$.

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