Wir 9: Exam 3 Review

Sections 15.1-15.4, 15.6-15.9

**Problem 1.** Let $R$ be the region in the $xy$-plane bounded by $y = 2x$, $x = 10$, and $y = -1$. Set up but do not evaluate $\int \int_R (x^2 + y^2) \, dA$ in the order $dy \, dx$ and $dx \, dy$.

**Problem 2.** Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} e^{-x^2-y^2} \, dy \, dx$.

**Problem 3.** Let $D$ be the region bounded by $y = 0$, $y = x^2$, and $x = 3$. Find $\int \int_D 3x \cos y \, dA$.

**Problem 4.** Compute $\int_0^3 \int_{3y}^9 7e^{x^2} \, dx \, dy$.

**Problem 5.** Let $R$ be the region that lies to the left of the $y$-axis between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$. Find $\int \int_R 5(x + y)$.

**Problem 6.** Find the volume of the solid that is above the $xy$ plane, below the ellipsoid $4x^2 + 4y^2 + z^2 = 64$ but inside the cylinder $x^2 + y^2 = 9$.

**Problem 7.** Let $D$ be the triangular region with vertices $(0, 1)$, $(1, 2)$, and $(4, 1)$. Set up but do not evaluate $\int \int_D 7y^2 \, dA$ in the order $dy \, dx$ and $dx \, dy$.

**Problem 8.** Let $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$. Evaluate $\int \int_D \frac{5y}{6x^5 + 1} \, dA$.

*With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.*
Problem 9. Express \( \int \int \int_E f(x,y,z) \, dV \) in the order \( dydzdx \) if \( E \) is the solid bounded by \( y = x^2, \quad z = 0, \quad y + 4z = 16 \).

Problem 10. Find the volume of the solid that is enclosed by the cylinder \( x^2 + y^2 = 9 \) and the planes \( y + z = 12 \) and \( z = 2 \).

Problem 11. Find the volume of the solid enclosed by the paraboloids \( y = x^2 + z^2 \) and \( y = 32 - x^2 - z^2 \).

Problem 12. Convert to Cylindrical: \( \int_{-9}^{9} \int_{-\sqrt{81-y^2}}^{\sqrt{81-y^2}} \int_{-\sqrt{y^2+z^2}}^{\sqrt{y^2+z^2}} xz \, dz \, dx \, dy \).

Problem 13. Find \( \int \int \int_E (x^2 + y^2 + z^2) \, dV \) where \( E \) is the part of the ball centered at the origin with radius 2 in the first octant.

Problem 14. Evaluate in spherical coordinates. \( \int_{0}^{10} \int_{0}^{\sqrt{100-x^2}} \int_{0}^{\sqrt{200-x^2-y^2}} yz \, dz \, dy \, dx \).

Problem 15. Let \( E \) be the region that lies between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 9 \). Set up but do not evaluate \( \int \int \int_E (x + y + z) \, dV \) in spherical coordinates.

Problem 16. Find the volume of the solid that lies within the sphere \( x^2 + y^2 + z^2 = 4 \), above the \( xy \) plane and below the cone \( z = \sqrt{x^2 + y^2} \).

Problem 17. Let \( R \) be the triangular region with vertices \( (0,0), \ (9,1), \ (1,9) \). Using the transformation \( x = 9u + v \) and \( y = u + 9v \) find \( \int \int_R (x - 10y) \, dA \).

Problem 18. Let \( R \) be the parallelogram enclosed by the lines \( x - 6y = 0 \), \( x - 6y = 9 \), \( 6x - y = 7 \), \( 6x - y = 10 \). Using the transformation \( u = x - 6y \) and \( v = 6x - y \), find \( \int \int_R \frac{9z-6y}{6x-9} \, dA \).

Problem 19. Let \( R \) be the region bounded by \( 25x^2 + 4y^2 = 100 \). Using the transformation \( x = 2u \) and \( y = 5v \), find \( \int \int_R 4x^2 \, dA \).

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