Section 2.2

Section 2.2 gives a method to solve differential equations can be written in the form:

$$f(y)\frac{dy}{dx} = g(x).$$

Since y is a function of x, the LHS looks like the derivative $\frac{d}{dx}F(y)$ of some function F. This is just the chain rule:

$$\frac{d}{dx}F(y) = F'(y)\frac{dy}{dx}.$$

So, $f(y) = F'(y)$, or, equivalently, $F(y) = \int f(y)dy$. So the first step in solving this equation is to find the anti–derivative of f. Then the LHS can be recognized as:

$$\frac{d}{dx}F(y) = g(x).$$

And so we have:

$$F(y) = \int g(x)dx + C,$$

and it is possible that we can solve this equation for y in terms of x.

Section 2.2

Problem 1. Find the general solution to $y' = x^2/y$.

Section 2.2

Problem 2. Find the general solution to $\frac{dy}{dx} = \frac{x-e^{-x}}{y+e^{y}}$.

Section 2.2

Problem 3. Solve the IVP:

$$y' = \frac{1-2x}{y}, \qquad y(1) = -2.$$

Plot the general solution and determine the interval on which the solution is defined.

Section 2.2

Section 2.2

Problem 4. Solve the IVP:

$$\frac{dy}{dx} = \frac{2x}{y + x^2 y}, \qquad y(0) = -2.$$

Plot the general solution and determine the interval on which the solution is defined.

Section 2.2

Section 2.2

Problem 5. Solve the IVP:

$$(\sin 2x)dx + (\cos 3y)dy = 0 \qquad y(\frac{\pi}{2}) = \frac{\pi}{3}.$$

Plot the general solution and determine the interval on which the solution is defined.

This is the same as:

$$\cos(3y)\frac{dy}{dx} = -\sin 2x.$$

Integrating both sides:

$$\int \cos(3y)dy = -\int \sin 2x dx$$

gives:

$$\frac{1}{3}\sin(3y) = \frac{1}{2}\cos(2x) + C.$$

Which can be written as:

$$2\sin(3y) - 3\cos(2x) = C.$$

Finding C:

$$2\sin(\pi) - 3\cos(\pi) = 3.$$

Thus:

$$2\sin(3y) - 3\cos(2x) = 3.$$

Solving:

$$\sin(3y) = \frac{3}{2}(1 + \cos(2x)) \Longrightarrow y = \frac{1}{3}\arcsin(\frac{3}{2}(1 + \cos(2x))) = \frac{1}{3}\arcsin(3\cos^2 x).$$

This isn't exactly correct because we need to make sure the argument to arcsin is between -1 and 1 and we need to specify which branch of arcsin to take.

Section 2.2

Section 2.3

Problem 6. Consider a tank used in an experiment. After one experiment, there is 200L of a dye solution with concentration of 1 gram per liter. The tank is rinsed with fresh water flowing in at 2 liters per minute and water is flowing out of the tank at the same rate. How long until the concentration is 1% of the original?

Section 2.3

Problem 7. A tank has 100 gallons of water and 50 oz of salt. Water containing salt with a concentration of $\frac{1}{4}(1 + \frac{1}{2}\sin t)$ ounces / gallon flows in at a rate of 2 gal / min and the mixture flows out at the same rate. Find the amount of salt at time t. What is $\lim S(t)$?