Section 2.4

Section 2.4 is about uniqueness and existence theorems. The first one is about first order linear equations:

Theorem 1. *If the functions p and g are continuous on an open interval $I = (\alpha, \beta)$ containing the point $t = t_0$, then there exists a unique function $x = \varphi(t)$ that solves the Initial Value Problem:*

$$x'(t) + p(t)x(t) = g(t), \qquad x(t_0) = x_0.$$

The next one is about more general first order equations.

Theorem 2. *Let the functions f and $\frac{\partial f}{\partial x}$ be continuous in some rectangle $\alpha < t < \beta$ and $\gamma < x < \delta$ that contains the point (t_0, x_0). Then in some interval $t_0 - h < t < t_0 + h$, contained in (α, β) there is a unique solution $x = \varphi(t)$ to the IVP:*

$$x' = f(t, x) \qquad x(t_0) = x_0.$$

Section 2.4

Problem 1. Determine – without solving the problem – an interval in which the solution of the given IVP is certain to exist:

$$(t-3)y' + (\ln t)y = 2t, \qquad y(1) = 2.$$

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Problem 2. Determine the region in the ty plane in which the hypotheses are the second theorem above are satisfied.

$$y' = \frac{\ln|ty|}{1 - t^2 + y^2}.$$

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Problem 3. Sketch several solution curves for the ODE:

$$y' = y(y-1)(y-2), y_0 \geq 0.$$

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Problem 4. Sketch several solution curves for the ODE:

$$y' = y^2(y^2 - 1), -\infty < y_0 < \infty.$$

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Problem 5. Sketch several solution curves for the ODE:

$$y' = y^2(1-y)^2, -\infty < y_0 < \infty.$$

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Section 2.6

If $F = F(x, y)$ and $y = y(x)$, then the multi variable chain rule says:

$$\frac{d}{dx}F(x, y) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx}.$$

Section 2.6

We want to study equations of the form:

$$M(x,y)+N(x,y)\frac{dy}{dx}=0.$$

The idea is that we want $M=F_x$ and $N=F_y$ for some function $F=F(x,y)$. If so, then this equation can be written:

$$\frac{d}{dx}F(x,y)=0,$$

whence a solution is:

$$F(x,y)=C,$$

and this defines y *implicitly* as a function of x.

So, how can we determine that $M=F_x$ and $N=F_y$. Recall that $F_{xy}=F_{yx}$, so if $M=F_x$ and $N=F_y$ then it is true that $M_y=N_x$ and, indeed, if M and N satisfy this equation, then they satisfy $M=F_x$ and $N=F_y$. In this case, we say the equation is *exact*.

Section 2.6

Determine if the equation is exact and find the solution:

$$(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0.$$

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Determine if the equation is exact and find the solution:

$$(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x) + (xe^{xy}\cos(2x) - 3)y' = 0.$$

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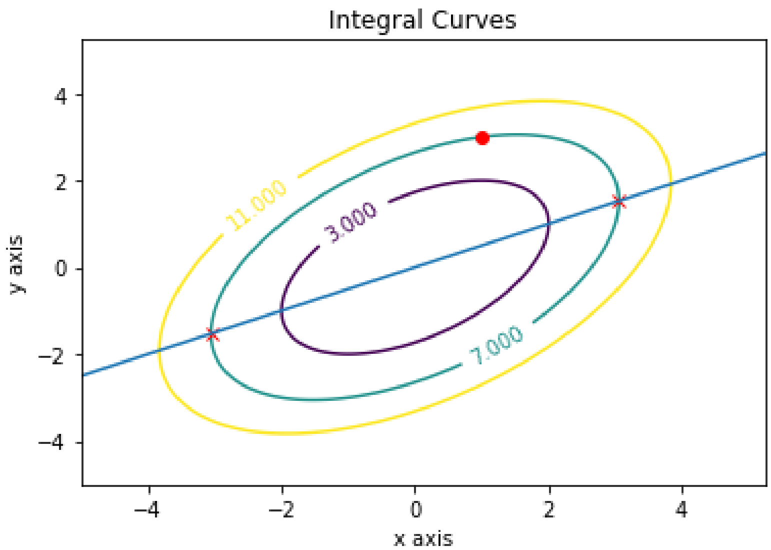
Find the solution (explicitly) to the IVP and the domain on which the solution is valid.

$$(2x - y) + (2y - x)y' = 0, \qquad y(1) = 3.$$

SECTION 2.6



The blue elipse is the one that corresponds to $y(1) = 3$ (this is the red dot). The red x's on the curve indicate the endpoints of the domain of validity. So the top blue curve is the curve that corresponds to the IVP. The line graphs the points where the tangent is vertical.

Section 2.6

Find an integrating factor to solve:

$$(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0.$$

Section 2.6