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When solving the ODE

$$ax'' + bx' + cx = 0,$$

find the roots to the characterstic polynomial $ar^2 + br + c$. There are three casese:

Roots are are real and distinct: General solution is $x(t) = c_1 e^{r_1 t} + c_2 e^{r_2 r}$.

Roots are complex conjudates: If the roots are $r_{1,2} = \mu \pm i\lambda$ then the general solution is $x(t) = c_1 e^{\mu t} \cos \lambda t + c_2 e^{\mu t} \sin \lambda t$.

Roots are real and repeated: If the polynomial is $(r - r_1)^2$ the general solution is $x(t) = c_1 e^{r_1 t} + c_2 t e^{r_2 t}$.

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Find the solution to $x'' - x' - 2x = 0$, $x(0) = 1$, $x'(0) = 1$.
$x(t) = \frac{2}{3}e^{2t} + \frac{1}{3}e^{-t}$.

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Find the solution to $x'' + 4x' + 3x = 0, x(0) = 2, x'(0) = -1$.
$x(t) = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$.

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Find the solution to $6x'' - 5x' + x = 0, x(0) = 4.x'(0) = 0$.
$x(t) = 12e^{\frac{t}{3}} - 8e^{\frac{t}{2}}$.

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Solve the IVP $y'' - 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$. The characteristic polynomial is $r^2 - 2r + 2 = 0$ and the roots are $r_{1,2} = 1 \pm i$. So the general solution is $y(t) = c_1 e^t \cos t + c_2 e^t \sin t$. Using the initial conditions, we get that $y(t) = e^t \cos t - e^t \sin t$.

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Solve the IVP $y'' - 2y' + 6y = 0$, $y(0) = 0$, $y'(0) = 1$. $x(t) = \frac{e^t \sin\sqrt{5}t}{\sqrt{5}}$.

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Solve the IVP $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = 2$. The characteristic polynomial is $r^2 + 2r + 2 = (r + 1)^2 + 1$ and so the roots are $r_{1,2} = -1 \pm i$. Thus, the general solution is $y(t) = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t$. To find the constants, use the initial condition, $1 = y(0) = c_1$ and since $2 = y'(t) = -c_1 e^{-t} \cos t - c_2 e^{-t} \sin t - c_2 e^{-t} \sin t + c_2 e^{-t} \cos t$, and $c_1 = 1$ we have $2 = y'(0) = -1 + c_2$ and so $c_2 = 3$. So the solution is $y(t) = e^{-t} \cos t + 3e^{-t} \sin t$.

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Solve the IVP $y'' - 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$. The characteristic polynomial is $r^2 - 2r + 1 = (r-1)^2$. There are repeated roots so the general solution is $y(t) = c_1 e^t + c_2 t e^t$. Using the initial condition let's us solve for c_1, c_2: $y(t) = e^t - te^t$.

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Solve the IVP $y'' - 2y' + 10y = 0$, $y(0) = 2$, $y'(0) = 0$. The characteristic polynomial is $r^2 - 2r + 10$ and the roots are $r_{1,2} = 1 \pm 3i$ and so the general solution is $x(t) = c_1 e^t \cos 3t + c_2 e^t \sin 3t$. Using the initial conditions gives $x(t) = 2e^t \cos 3t - \frac{2}{3} e^t \sin 3t$.

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Solve the IVP $x'' - 5x' + 6x = 2e^t$, $x(0) = 1$, $x'(0) = 0$.

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Solve the IVP $x'' + 2x' + x = 3e^{-t}$, $x(0) = 1$, $x'(0) = 1$.

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Solve the IVP $x'' + 2x' + x = \cos(2t)$, $x(0) = 1$, $x'(0) = 1$.

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