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When solving the ODE

\[ ax'' + bx' + cx = 0, \]

find the roots to the characteristic polynomial \( ar^2 + br + c \). There are three cases:

Roots are real and distinct: General solution is \( x(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} \).

Roots are complex conjugates: If the roots are \( r_1, r_2 = \mu \pm i\lambda \) then the general solution is \( x(t) = c_1 e^{\mu t} \cos \lambda t + c_2 e^{\mu t} \sin \lambda t \).

Roots are real and repeated: If the polynomial is \( (r - r_1)^2 \) the general solution is \( x(t) = c_1 e^{r_1 t} + c_2 te^{r_1 t} \).
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Find the solution to $x'' - x' - 2x = 0$, $x(0) = 1$, $x'(0) = 1$.

$x(t) = \frac{2}{3}e^{2t} + \frac{1}{3}e^{-t}$. 
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Find the solution to $x'' + 4x' + 3x = 0, x(0) = 2, x'(0) = -1.$

$x(t) = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}.$
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Find the solution to $6x'' - 5x' + x = 0, x(0) = 4, x'(0) = 0.$
$x(t) = 12e^{\frac{t}{2}} - 8e^{\frac{t}{2}}.$
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Solve the IVP $y'' - 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$. The characteristic polynomial is $r^2 - 2r + 2 = 0$ and the roots are $r_{1,2} = 1 \pm i$. So the general solution is $y(t) = c_1 e^t \cos t + c_2 e^t \sin t$. Using the initial conditions, we get that $y(t) = e^t \cos t - e^t \sin t$. 

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Solve the IVP \( y'' - 2y' + 6y = 0, \quad y(0) = 0, \quad y'(0) = 1. \quad x(t) = \frac{e^t \sin \sqrt{5} t}{\sqrt{5}}. \)
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Solve the IVP \( y'' + 2y' + 2y = 0 \), \( y(0) = 1 \), \( y'(0) = 2 \). The characteristic polynomial is \( r^2 + 2r + 2 = (r + 1)^2 + 1 \) and so the roots are \( r_{1,2} = -1 \pm i \). Thus, the general solution is \( y(t) = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t \). To find the constants, use the initial condition, \( 1 = y(0) = c_1 \) and since \( 2 = y'(t) = -c_1 e^{-t} \cos t - c_2 e^{-t} \sin t - c_2 e^{-t} \sin t + c_2 e^{-t} \cos t \), and \( c_1 = 1 \) we have \( 2 = y'(0) = -1 + c_2 \) and so \( c_2 = 3 \). So the solution is \( y(t) = e^{-t} \cos t + 3e^{-t} \sin t \).
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Solve the IVP \( y'' - 2y' + y = 0, \ y(0) = 1, \ y'(0) = 0. \) The characteristic polynomial is 
\[ r^2 - 2r + 1 = (r - 1)^2. \] There are repeated roots so the general solution is 
\[ y(t) = c_1 e^t + c_2 te^t. \] Using the initial condition let’s us solve for \( c_1, c_2: \ y(t) = e^t - te^t. \)
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Solve the IVP $y'' - 2y' + 10y = 0$, $y(0) = 2$, $y'(0) = 0$. The characteristic polynomial is $r^2 - 2r + 10$ and the roots are $r_{1,2} = 1 \pm 3i$ and so the general solution is $x(t) = c_1 e^t \cos 3t + c_2 e^t \sin 3t$. Using the initial conditions gives $x(t) = 2e^t \cos 3t - \frac{2}{3}e^t \sin 3t$. 
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Solve the IVP $x'' - 5x' + 6x = 2e^t$, $x(0) = 1$, $x'(0) = 0$. 
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Solve the IVP $x'' + 2x' + x = 3e^{-t}$, $x(0) = 1$, $x'(0) = 1$. 
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Solve the IVP $x'' + 2x' + x = \cos(2t), \ x(0) = 1, \ x'(0) = 1$. 
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