Problem 1^4.

In 2013, the Pew Research Foundation reported that 45% of U.S. adults report that they live with one or more chronic conditions^1. However, this value was based on a sample, so it may not be a perfect estimate for the population parameter of interest on its own. The standard deviation of about 1.2%, and a normal model may reasonably be used in this setting.

1. Which conditions were checked in order to use the normal model?
   \[ np \hat{p} \geq 10 \quad \text{and} \quad n(1-\hat{p}) \geq 10 \]

2. Create a 95% confidence interval for the proportion of U.S. adults who live with one or more chronic conditions. Also interpret the confidence interval in the context of the study.

Answer: Recall that the general formula is \[ \text{point estimate} \pm Z^* \times \text{SE} \]

First, identify the three different values. The point estimate is 45%, \( Z^* = 1.96 \) for a 95% confidence level, and \( Sd = 1.2\% \). Then, plug the values into the formula:

\[ 45\% \pm 1.96 \times 1.2\% \rightarrow (42.6\%, 47.4\%) \]

We are 95% confident that the proportion of US adults who live with one or more chronic conditions is between 42.6% and 47.4%.

Problem 2

A hospital administrator hoping to improve wait times decides to estimate the average emergency room waiting time at her hospital. She collects a simple random sample of 64 patients and determines the time (in minutes) between when they checked in to the ER until they were first seen by a doctor. A 95% confidence interval based on this sample is (128 minutes, 147 minutes), which is based on the normal model for the mean.

Determine whether the following statements are true or false, and explain your reasoning. ^4

3. This confidence interval is not valid since we do not know if the population distribution of the ER wait times is nearly Normal.
   False, because \( n \geq 30 \)

4. We are 95% confident that the average waiting time of these 64 emergency room patients is between 128 and 147 minutes.
   False, because inference is made on parameter

5. We are 95% confident that the average waiting time of all patients at this hospital’s emergency room is between 128 and 147 minutes.
   True

6. 95% of random samples have a sample mean between 128 and 147 minutes.
   False, the CI is not about a sample mean.

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^1 Math-UOttawa 2. UVermont 3 Utts ^4 OpenIntro
7. A 99% confidence interval would be narrower than the 95% confidence interval since we need to be surer of our estimate.
   False, it will be wider
8. The margin of error is 9.5 and the sample mean is 137.5.
   True
9. In order to decrease the margin of error of a 95% confidence interval to half of what it is now, we would need to double the sample size.
   False, To cut the SE (or margin of error) in half, we would need a sample of 256 that is four times the original sample size.

Problem 1

In 2013, the Pew Research Foundation reported that “45% of U.S. adults report that they live with one or more chronic conditions”, and the standard deviation is 1.2%. Identify each of the following statements as true or false. Provide an explanation to justify each of your answers.

10. We can say with certainty that the confidence interval from Problem 1 contains the true percentage of U.S. adults who suffer from a chronic illness.
   Answer
   (a) False. Confidence intervals provide a range of plausible values, and sometimes the truth is missed. A 95% confidence interval “misses” about 5% of the time.
   (b) True. Notice that the description focuses on the true population value.
   (c) False. The standard deviation describes the uncertainty in the overall estimate from natural fluctuations due to randomness, not the uncertainty corresponding to individuals’ responses.

Problem 3

The 2010 General Social Survey asked the question: “After an average work day, about how many hours do you have to relax or pursue activities that you enjoy?” to a random sample of 1,155 Americans. A 95% confidence interval for the mean number of hours spent relaxing or pursuing activities they enjoy was (1.38, 1.92).

13. Interpret this interval in context of the data.
14. Suppose another set of researchers reported a confidence interval with a larger margin of error based on the same sample of 1,155 Americans. How does their confidence level compare to the confidence level of the interval stated above?
15. Suppose next year a new survey asking the same question is conducted, and this time the sample size is 2,500. Assuming that the population characteristics, with respect to how much time people spend relaxing after work, have not changed much within a year. How will the margin of error of the

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1 Math-UOttawa 2. UVermont 3 Utts 4 OpenIntro
95% confidence interval constructed based on data from the new survey compare to the margin of error of the interval stated above?

Answers:
(a) We are 95% confident that Americans spend an average of 1.38 to 1.92 hours per day relaxing or pursuing activities they enjoy.
(b) Their confidence level must be higher as the width of the confidence interval increases as the confidence level increases.
(c) The new margin of error will be smaller since as the sample size increases the standard error decreases, will decrease the margin of error.

Problem 4

A hospital administrator hoping to improve wait times decides to estimate the average emergency room waiting time at her hospital. She collects a simple random sample of 64 patients and determines the time (in minutes) between when they checked in to the ER until they were first seen by a doctor. A 95% confidence interval based on this sample is (128 minutes, 147 minutes), which is based on the normal model for the mean. Determine whether the following statements are true or false, and explain your reasoning.

16. This confidence interval is not valid since we do not know if the population distribution of the ER wait times is nearly Normal.
17. We are 95% confident that the average waiting time of these 64 emergency room patients is between 128 and 147 minutes.
18. We are 95% confident that the average waiting time of all patients at this hospital’s emergency room is between 128 and 147 minutes.
19. 95% of random samples have a sample mean between 128 and 147 minutes.
20. A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate.
21. The margin of error is 9.5 and the sample mean is 137.5.
22. In order to decrease the margin of error of a 95% confidence interval to half of what it is now, we would need to double the sample size.

Problem 5

A survey conducted on a reasonably random sample of 203 undergraduates asked, among many other questions, about the number of exclusive relationships these students have been in. The histogram below shows the distribution of the data from this sample. The sample average is 3.2. The standard deviation of was previously reported as 1.97.

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Estimate the average number of exclusive relationships Duke students have been in using a 90% confidence interval and interpret this interval in context. Check any conditions required for inference, and note any assumptions you must make as you proceed with your calculations and conclusions.

Answer: Independence: random sample. We can assume that the students in this sample are independent of each other with respect to number of exclusive relationships they have been in. Notice that there are no students who have had no exclusive relationships in the sample, which suggests some student responses are likely missing (perhaps only positive values were reported). The sample size is at least 30. The skew is strong, but the sample is large so this is not a concern. 90% CI: (2.97, 3.43). We are 90% confident that undergraduate students have been in 2.97 to 3.43 exclusive relationships, on average.