SECTION 5.5: PIECEWISE-DEFINED FUNCTIONS

Pr 1. Given $f(x) = \begin{cases} 3x^2 \ (A) & \text{if} \quad x < -4 \\ \sqrt{x+4} \ (B) & \text{if} \quad -4 \le x < 9 \\ \frac{x}{x+1} \ (C) & \text{if} \quad x > 9 \end{cases}$, determine each of the following:

(a) $f(-6)$

$-6 < -4$ so use $3x^2$

$f(-6) = 3(-6)^2$
$= 3 \cdot 36$
$= 108$

(b) $f(-4)$

-4 is in the interval $-4 \le x < 9$ so use $\sqrt{x+4}$

$f(-4) = \sqrt{(-4)+4}$
$= \sqrt{0}$
$= 0$

(c) $f(0)$

0 is in the interval $-4 \le x < 9$ so use $\sqrt{x+4}$

$f(0) = \sqrt{(0)+4}$
$= \sqrt{4}$
$= 2$

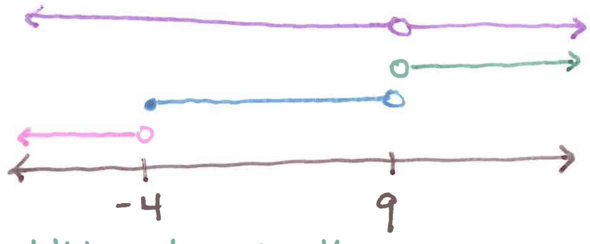
(d) $f(9)$

9 is not in any intervals

$f(9)$ does not exist

Pr 2. State the domain, using interval notation, of each of the following

(a) $f(x) = \begin{cases} 3x^2 & \text{if} \quad x < -4 \quad \text{(A)} \\ \sqrt{x+4} & \text{if} \quad -4 \le x < 9 \quad \text{(B)} \\ \frac{x}{x+1} & \text{if} \quad x > 9 \quad \text{(C)} \end{cases}$



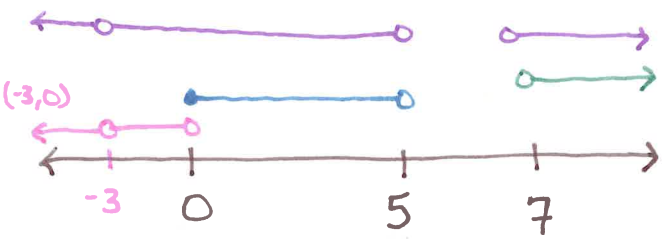
A) $3x^2$ - no additional restrictions

B) $\sqrt{x+4} \Rightarrow x \geq -4$ so no additional restrictions

C) $\frac{x}{x+1} \Rightarrow x \neq -1$, not in stated interval, so no additional restrictions

Domain: $x \in (-\infty, 9) \cup (9, \infty)$

(b) $g(x) = \begin{cases} \frac{x+6}{x+3} & \text{if} \quad x < 0 \quad \text{(A)} \\ 7 - x^2 & \text{if} \quad 0 \le x < 5 \quad \text{(B)} \\ \sqrt[3]{x-8} & \text{if} \quad x > 7 \quad \text{(C)} \end{cases}$



A) $\frac{x+6}{x+3} \Rightarrow x \neq -3$ so new intervals $(-\infty,-3)\cup(-3,0)$

B) $7-x^2$ - no additional restrictions

C) $\sqrt[3]{x-8}$ - no additional restriction

Domain: $x \in (-\infty,-3) \cup (-3,5) \cup (7,\infty)$

Pr 3. State the equivalent piecewise-defined function for the absolute value function, $h(x) = -5|8 - 3x|$.

$$h(x) = \begin{cases} -[-5(8-3x)] & \text{if} \quad 8-3x < 0 \\ [-5(8-3x)] & \text{if} \quad 8-3x \geq 0 \end{cases}$$

$8-3x<0$
$-3x<-8$
$x>8/3$

$8-3x\geq 0$
$-3x \geq -8$
$x \leq 8/3$

$$h(x) = \begin{cases} 5(8-3x) & \text{if} \quad x > 8/3 \\ -5(8-3x) & \text{if} \quad x \leq 8/3 \end{cases} \quad \text{or} \quad h(x) = \begin{cases} -5(8-3x) & \text{if} \quad x \leq \frac{8}{3} \\ 5(8-3x) & \text{if} \quad x > 8/3 \end{cases}$$

Pr 4. State the equivalent absolution value function for the piecewise-defined function, $f(x) = \begin{cases} -(2x-5) & \text{if} \quad x < \frac{5}{2} \\ 2x-5 & \text{if} \quad x \geq \frac{5}{2} \end{cases}$

$f(x) = |2x-5|$

Pr 5. Write the corresponding piecewise-defined function for the scenario: A cleaning service charges \$20 an hour plus a \$25 service fee for the first three hours of cleaning. For any job that requires more than three hours, the price increases to \$30 per additional hour, but the service fee is waived.

$$C(x) = \begin{cases} \text{first three hours} \\ \text{more than 3 hours} \end{cases}$$

$$C(h) = \begin{cases} 20h + 25 & \text{if } 0 \le h \le 3 \\ \underbrace{30(h-3)}_{\text{additional hours}} + \underbrace{20(3)}_{\text{first 3 hours}} & h > 3 \end{cases}$$

$$C(h) = \begin{cases} 20h + 25 & \text{if } 0 \le h \le 3 \\ 30(h-3) + 60 & \text{if } h > 3 \end{cases}$$

Pr 1. Rewrite each exponential expression as a single equivalent expression in the stated base.

(a) $27 \cdot 9^{x-1}$, base 3

$27 = 3^3$

$9 = 3^2$

$$27 \cdot 9^{x-1} = 3^3 \cdot (3^2)^{x-1}$$
$$= 3^3 \cdot 3^{2x-2}$$
$$= 3^{3+2x-2}$$
$$= 3^{2x+1}$$

(b) $\left(\frac{1}{4}\right)^x \cdot \left(\frac{8}{2^{x+4}}\right)$, base 2

$\frac{1}{4} = 2^{-2}$

$8 = 2^3$

$$\left(\frac{1}{4}\right)^x \cdot \left(\frac{8}{2^{x+4}}\right) = (2^{-2})^x \cdot \left(\frac{2^3}{2^{x+4}}\right)$$
$$= 2^{-2x} \cdot 2^{3-(x+4)}$$
$$= 2^{-2x+(-x-1)}$$
$$= 2^{-3x-1}$$

Pr 2. Determine if each function is an exponential function. If the function is an exponential function, determine whether the function represents exponential growth or decay.

(a) $f(x) = 3^{-x}$

$f(x) = \left(\frac{1}{3}\right)^x$

Exponential decay

$b = \frac{1}{3}$

(b) $g(x) = -4x^5$

polynomial, not an exponential function

(c) $h(x) = \frac{2}{9}e^{x+3}$

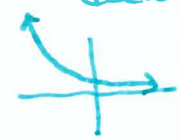
exponential growth

$b = e$

$$h(x) = \frac{2}{9} e^x \cdot e^3$$
$$= \frac{2e^3}{9} \cdot e^x$$

Pr 3. State the domain, range, end behavior, x-intercept(s), and y-intercept of $g(x) = 7^{2-x}$.

$$g(x) = 7^2 \cdot 7^{-x}$$
$$= 49 \cdot \underbrace{\left(\tfrac{1}{7}\right)^x}_{\text{exponential decay}}$$



Domain

$x \in (-\infty, \infty)$

Range

$y \in (0, \infty)$

End behavior

As $x \to -\infty$, $g(x) \to \infty$

As $x \to \infty$, $g(x) \to 0$

x-intercept(s)

None as $0 \neq 7^{2-x}$

y-intercept

$$g(0) = 7^{2-0}$$
$$= 7^2$$
$$= 49$$

$(0, 49)$

Pr 4. State the domain of the function, using interval notation.

(a) $f(x) = 10^{\frac{3x}{x-8}}$

Domain:
$(-\infty, 8) \cup (8, \infty)$

$f(x)$ is defined when $\frac{3x}{x-8}$ is defined

$\frac{3x}{x-8}$ is defined when $x-8 \neq 0$

$x-8 \neq 0$
$x \neq 8$

(b) $g(x) = e^{\sqrt[4]{3-9x}}$

Domain:
$(-\infty, \frac{1}{3}]$

$g(x)$ is defined when $\sqrt[4]{3-9x}$ is defined

$\sqrt[4]{3-9x}$ is an even root, so $3-9x \geq 0$

$3-9x \geq 0$
$-9x \geq -3$
$x \leq \frac{1}{3}$

(c) $h(x) = \dfrac{\sqrt[7]{4x+11}}{2^{3x-7}}$

Domain:
$(-\infty, \infty)$

$h(x)$ is defined when both the numerator and denominator are defined

Numerator: $\sqrt[7]{4x+11}$ is odd root so no restrictions

Denominator: 2^{3x-7} is defined when $3x-7$ is defined, $3x-7$ is a polynomial and has no restrictions

AND $2^{3x-7} \neq 0$

2^{3x-7} is always positive so no restrictions

Pr 5. Algebraically solve each equation for x.

(a) $\left(\frac{1}{3}\right)^{2x} = 9^{x-6}$

$$\left(\frac{1}{3}\right)^{2x} = 9^{x-6}$$

$$(3^{-1})^{2x} = (3^2)^{x-6}$$

$$3^{-2x} = 3^{2x-12}$$

so

$$-2x = 2x-12$$

$$-4x = -12$$

$$\boxed{x = 3}$$

(b) $e^x\left(e^x + e\right) = \dfrac{e^x + e^{3x}}{e^{-x}}$

$$e^x(e^x + e) = \frac{e^x + e^{3x}}{e^{-x}}$$

$$e^{2x} + e^{x+1} = e^{x-(-x)} + e^{3x-(-x)}$$

$$e^{2x} + e^{x+1} = e^{2x} + e^{4x}$$

$$e^{x+1} = e^{4x}$$

so

$$x+1 = 4x$$

$$1 = 3x$$

$$\boxed{\frac{1}{3} = x}$$

(c) $\left(\frac{1}{100}\right)^{-x^2} \cdot 10^{4x} - 1 = 0$

$$\left(\frac{1}{100}\right)^{-x^2} \cdot 10^{4x} - 1 = 0$$

$$(10^{-2})^{-x^2} \cdot 10^{4x} - 1 = 0$$

$$10^{2x^2} \cdot 10^{4x} - 1 = 0$$

$$10^{2x^2+4x} - 1 = 0$$

$$10^{2x^2+4x} = 1$$

$$10^{2x^2+4x} = 10^0$$

so

$$2x^2 + 4x = 0$$

$$2x(x+2) = 0$$

$$2x = 0 \text{ or } x+2 = 0$$

$$\boxed{x = 0} \text{ or } \boxed{x = -2}$$

Pr 6. If you invest \$2000 in an account that earns interest at a rate of 3.16% per year, compounded monthly, how much will be in the account after 10 years? If the annual interest is compounded continuously instead of monthly, how much more will be in the account after 10 years compared to your previous answer?

$A = P\left(1+\frac{r}{m}\right)^{mt}$

$P = 2000$

$r = 0.0316$

$m = 12$

$t = 10$

$A = 2000\left(1 + \frac{0.0316}{12}\right)^{12(10)}$

$A \approx \$2742.12$

monthly

$A = Pe^{rt}$

$P = 2000$

$r = 0.0316$

$t = 10$

$A = 2000e^{0.0316(10)}$

$A \approx \$2743.26$

continuously

you earn \$1.14 more in 10 years if interest is compounded continuously.

Pr 7. If company opens in 2017, and the company's revenue grows at an annual rate of 150% per year, the revenue function would be $R(t) = R_0\left(\frac{3}{2}\right)^t$, where R_0 represents the initial revenue earned in 2017, and t represents the number of years since 2017. How much money did the company bring in, in revenue, in 2017, if the company's revenue is \$864,000 in 2020?

$R(3)$ $t=3$

$R(t) = R_0\left(\frac{3}{2}\right)^t$

$R(3) = R_0\left(\frac{3}{2}\right)^3$

$864000 = \frac{27}{8}R_0$

$256000 = R_0$

In 2017, the company's revenue was \$256,000.

Pr 1. Given $f(x) = 1 - x^2$, $g(x) = 4 \cdot 6^x$, and $h(x) = \dfrac{2x}{1-x}$, calculate each operation and simplify, if possible.

$x \neq 1$

(a) $(f+g)(2)$

$$(f+g)(2) = f(2) + g(2)$$
$$= [1-(2)^2] + [4 \cdot 6^{(2)}]$$
$$= -3 + 144$$
$$= 141$$

(b) $(fh)(4)$

$$(fh)(4) = f(4) \cdot h(4)$$
$$= [1-(4)^2] \cdot \left[\frac{2(4)}{1-(4)}\right]$$
$$= (-15)\left(-\frac{8}{3}\right)$$
$$= 40$$

(c) $\left(\dfrac{h}{f}\right)(3)$

$$\left(\frac{h}{f}\right)(3) = \frac{h(3)}{f(3)}$$
$$= \frac{\left[\frac{2(3)}{1-(3)}\right]}{1-(3)^2}$$
$$= \frac{-3}{-8}$$
$$= \frac{3}{8}$$

(d) $(g-h)(x)$

$$(g-h)(x) = g(x) - h(x)$$
$$= 4 \cdot 6^{(x)} - \frac{2(x)}{1-(x)}$$
$$= 4 \cdot 6^x - \frac{2x}{1-x}$$

Pr 2. Given $f(x) = 1 - x^2$, $g(x) = 4 \cdot 6^x$, and $h(x) = \frac{2x}{1-x}$, calculate each operation and simplify, if possible.

(a) $(f \circ h)(4)$

$$(f \circ h)(4) = f(h(4))$$
$$= f\left(\frac{2(4)}{1-(4)}\right)$$
$$= f\left(-\frac{8}{3}\right)$$
$$= 1 - \left(-\frac{8}{3}\right)^2$$
$$= -55/9$$

(b) $g(h(0))$

$$g(h(0)) = g(h(0))$$
$$= g\left(\frac{2(0)}{1-(0)}\right)$$
$$= g(0)$$
$$= 4 \cdot 6^{(0)}$$
$$= 4 \cdot 1 = 4$$

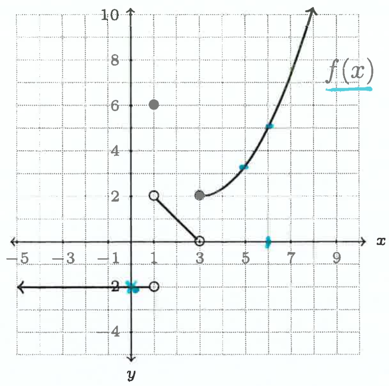
(c) $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$
$$= f(4 \cdot 6^{(x)})$$
$$= 1 - (4 \cdot 6^x)^2$$
$$= 1 - 16 \cdot 6^{2x}$$

(d) $(f \circ f)(x)$

$$(f \circ f)(x) = f(f(x))$$
$$= f(1 - (x)^2)$$
$$= f(1 - x^2)$$
$$= 1 - (1 - x^2)^2$$
$$= 1 - [1 - 2x^2 + x^4]$$
$$= 2x^2 - x^4$$

Pr 3. Use the graph and table provided to calculate each operation and simplify, if possible.



x	$g(x)$
-2	6
-1	-5
0	1
1	3
4	-3
6	12

(a) $(f-g)(6)$

$$(f-g)(6) = f(6) - g(6)$$
$$= 5 - 12$$
$$= -7$$

(b) $(g \circ f)(0)$

$$(g \circ f)(0) = g(f(0))$$
$$= g(-2)$$
$$= 6$$

(c) $f(f(g(-2)))$

$$f(f(g(-2))) = f(f(g(-2)))$$
$$= f(f(6))$$
$$= f(5)$$
$$= 3\tfrac{1}{3}$$