SECTION 5.8: INVERSE FUNCTIONS AND LOGARITHMS

Pr 1. Rewrite each exponential expression as an equivalent logarithmic expression.

(a) $9^{x-1} = 245$

$\log_9(245) = x - 1$

(b) $10^{2x-7} = \dfrac{3}{1000}$

$\log_{10}\left(\frac{3}{1000}\right) = 2x - 7$

Pr 2. Rewrite each logarithmic expression as an equivalent exponential expression.

(a) $\log_{\frac{5}{4}}(5x^2) = 8$

$\left(\frac{5}{4}\right)^8 = 5x^2$

(b) $\ln(x) = 4$

$e^4 = x$

Pr 3. State the domain of each algebraic function, using interval notation.

(a) $f(x) = \log_5(5x - 2)$

$f(x)$ is defined when $5x - 2 > 0$

$5x > 2$

$x > \frac{2}{5}$

Domain: $\left(\frac{2}{5}, \infty\right)$

(b) $g(x) = \log\left(\sqrt{8x+1}\right)$

$g(x)$ is defined when $\sqrt{8x+1} > 0$ AND $8x + 1 \geq 0$

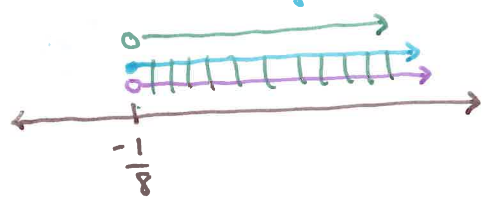
Domain: $\left(-\frac{1}{8}, \infty\right)$

$\sqrt{8x+1} > 0$ AND $8x + 1 \geq 0$

$8x + 1 > 0$ $\quad$ $8x \geq -1$

$8x > -1$ $\quad$ $x \geq -\frac{1}{8}$

$x > -\frac{1}{8}$



(c) $h(x) = \dfrac{\ln(6 - x)}{x^2 + 4x + 4}$

$h(x)$ is defined when both the numerator and denominator are defined

Domain: $(-\infty, -2) \cup (-2, 6)$

Numerator: $\ln(6-x)$ is defined when

$6 - x > 0$

$-x > -6$

$x < 6$

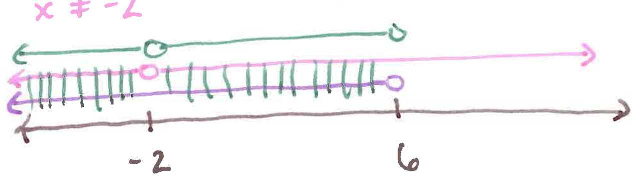
Denominator: $x^2 + 4x + 4$ has no restrictions

AND $x^2 + 4x + 4 \neq 0$

$(x+2)(x+2) \neq 0$

$x + 2 \neq 0$ AND $x + 2 \neq 0$

$x \neq -2$ $\quad$ $x \neq -2$



Pr 4. Simplify $\ln\left(e^{4x-13}\right)$.

$$\ln\left(e^{4x-13}\right) = (4x-13)\ln(e)$$
$$= 4x-13$$

Pr 5. Express each expression as a single logarithm. Assume when necessary that all variables/expressions represent positive real numbers.

(a) $3\log(x) - \log(x+1) - \log(x-1)$

$$3\log(x) - \log(x+1) - \log(x-1) = \log(x^3) - \log(x+1) - \log(x-1)$$
$$= \log\left[\frac{x^3}{(x+1)(x-1)}\right]$$

(b) $\frac{1}{2}(-\ln(x-2) + 5\ln(3x+8) + 2\ln(8-x))$

$$\frac{1}{2}\left(-\ln(x-2) + 5\ln(3x+8) + 2\ln(8-x)\right) = \frac{1}{2}\left(-\ln(x-2) + \ln\left((3x+8)^5\right) + \ln\left((8-x)^2\right)\right)$$
$$= \frac{1}{2}\ln\left[\frac{(3x+8)^5(8-x)^2}{x-2}\right]$$
$$= \ln\left[\left(\frac{(3x+8)^5(8-x)^2}{x-2}\right)^{1/2}\right]$$

Pr 6. Use the properties of logarithms to fully expand and simplify each expression. Assume when necessary that all variables/expressions represent positive real numbers.

(a) $\log_b\left(\sqrt{\frac{x-2}{x}}\right) = \log_b\left[\left(\frac{x-2}{x}\right)^{1/2}\right]$

$= \frac{1}{2}\log_b\left(\frac{x-2}{x}\right)$

$= \frac{1}{2}\left[\log_b(x-2) - \log_b(x)\right]$

(b) $\ln\left(\frac{\sqrt{3-x}}{3x(x-7)}\right) = \ln\left(\frac{(3-x)^{1/2}}{3x(x-7)}\right)$

$= \ln\left((3-x)^{1/2}\right) - \ln(3x) - \ln(x-7)$

$= \frac{1}{2}\ln(3-x) - \ln(3) - \ln(x) - \ln(x-7)$

Pr 7. Algebraically solve each equation for x.

(a) $32^x = 18^{x+1}$

$32 = 2^5$, $18 = 2 \cdot 3^2$ > not like bases

$32^x = 18^{x+1}$

$\ln(32^x) = \ln(18^{x+1})$

$x\ln(32) = (x+1)\ln(18)$

$x\ln(32) = x\ln(18) + \ln(18)$

$x\ln(32) - x\ln(18) = \ln(18)$

$x(\ln(32) - \ln(18)) = \ln(18)$ $\rightarrow$ $x = \frac{\ln(18)}{\ln(32) - \ln(18)}$

(b) $(e^{x^2} - 4)(e^x + 1) = 0$

$e^{x^2} - 4 = 0$ or $e^x + 1 = 0$

$e^{x^2} = 4$ $\qquad$ $\underbrace{e^x = -1}_{\text{Never}}$

$\ln(e^{x^2}) = \ln(4)$

$x^2\ln(e) = \ln(4)$

$x^2 = \ln(4)$

$x = \pm\sqrt{\ln(4)}$

Pr 8. Algebraically solve each equation for x.

(a) $3\ln(x+4) - 5 = 8$ Domain: $x > -4$

$3\ln(x+4) - 5 = 8$

$3\ln(x+4) = 13$

$\ln(x+4) = \frac{13}{3}$

$e^{13/3} = x+4$

$e^{13/3} - 4 = x$

(b) $\log(\underset{x>-3}{x+3}) + \log(\underset{x>1}{x-1}) = \log(\underset{x<15/2}{15-2x})$ Domain: $1 < x < \frac{15}{2}$

$\log(x+3) + \log(x-1) = \log(15-2x)$

$\log(x^2+2x-3) = \log(15-2x)$

so

$x^2+2x-3 = 15-2x$

$x^2+4x-18 = 0$

$x = \frac{-4 \pm \sqrt{16-4(1)(-18)}}{2}$

$x = \frac{-4 \pm \sqrt{88}}{2} \approx -6.6904$ or 2.6904 → not in domain

$x = \frac{-4+\sqrt{88}}{2}$ only

Pr 9. If you invest \$2000 in an account that earns interest at a rate of 3.16% per year, compounded continuously, how long will it take for the amount in the account to triple?

$A = Pe^{rt}$

$A = 6000 = 3(2000)$

$P = 2000$

$r = 0.0316$

$t = ?$

$A = Pe^{rt}$

$6000 = 2000\, e^{0.0316t}$

$3 = e^{0.0316t}$

$\ln(3) = \ln(e^{0.0316t})$

$\ln(3) = 0.0316t$

$\frac{\ln(3)}{0.0316} = t$

$34.7662 \approx t$

Exact Answer: $\frac{\ln(3)}{0.0316}$ yrs Rounded Answer: 34.7662 yrs