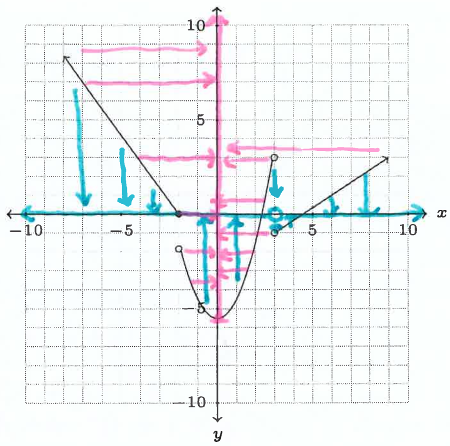
EXAM 3 REVIEW OVER CHAPTER 5

Pr 1. State the domain and range of the function given in the graph below, using interval notation.



Domain: $x \in (-\infty, 3) \cup (3, \infty)$

Range: $y \in [-5.5, \infty)$

Pr 2. Determine if $h(x) = 8x^5 - x^2 + 26x^7 - 60$ is a polynomial function or not. If the function is a polynomial, state the degree, leading coefficient, and constant term.

$h(x) = 8x^5 - x^2 + 26x^7 - 60$

Leading term

A polynomial function

Degree: 7

Leading Coefficient: 26

Constant Term: −60

Pr 3. State the domain, range, vertex, y-intercept, and any x-intercepts for $f(x) = -4x^2 + 32x - 28$

Domain: $x \in (-\infty, \infty)$

Range: $y \in (-\infty, 36]$

Vertex: $(4, 36)$

y-intercept: $(0, -28)$

x-intercept(s): $(1, 0)$
$(7, 0)$

Vertex:
$x = -\frac{b}{2a} = \frac{-(32)}{2(-4)} = 4$

$f(-\frac{b}{2a}) = -4(4)^2 + 32(4) - 28$
$= 36$

Since $a < 0$ ↷

y-intercept:
$f(0) = -4(0)^2 + 32(0) - 28$
$= -28$

x-intercept(s):
$0 = -4x^2 + 32x - 28$
$0 = x^2 - 8x + 7$
$0 = (x-1)(x-7)$
$x - 1 = 0$ or $x - 7 = 0$
$x = 1$ $x = 7$

Pr 4. The price-demand function (in dollars) for a particular item is given by $p(x) = -0.08x + 68$, where x is the number of items. The company who produces these items has a production cost of \$4 per item and fixed costs of \$160. Determine the maximum profit for the company from the sales of this item.

Vertex P(x) need y-value

price: $p(x) = -0.08x + 68$

Revenue: $R(x) = \text{price} \cdot \text{quantity}$
$R(x) = (-0.08x + 68)x$
$R(x) = -0.08x^2 + 68x$

Total Cost: $C(x) = \text{production cost} \cdot \text{quantity} + \text{fixed costs}$
$C(x) = 4x + 160$

Profit: $P(x) = \text{Revenue} - \text{Total Cost}$
$P(x) = (-0.08x^2 + 68x) - (4x + 160)$
$P(x) = -0.08x^2 + 64x - 160$

Vertex of P(x)
$x = \frac{-b}{2a}$
$x = \frac{-(64)}{2(-0.08)} = 400$

$P(400) = -0.08(400)^2 + 64(400) - 160$
$= 12640$

The maximum profit for the company is \$12,640

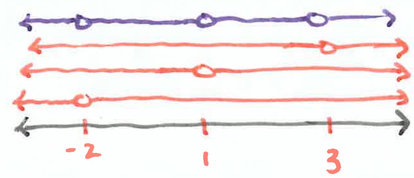
Pr 5. State the domain, y-intercept (if it exists), any x-intercepts, any vertical asymptotes, and any holes for the rational function, $f(x) = \dfrac{x(x-3)(x+4)}{(x-1)(x-3)(x+2)}$. $= \dfrac{x(x+4)}{(x-1)(x+2)}, \; x \neq 3$

Domain: $x \in (-\infty, -2) \cup (-2, 1) \cup (1, 3) \cup (3, \infty)$

$(x-1)(x-3)(x+2) \neq 0$

$x-1 \neq 0$ AND $x-3 \neq 0$ AND $x+2 \neq 0$

$x \neq 1$ AND $x \neq 3$ AND $x \neq -2$



y-intercept: $(0, 0)$

$$f(0) = \frac{(0)[(0)+4]}{[(0)-1][(0)+2]} = 0$$

x-intercept(s): $(0, 0)$ $(-4, 0)$

$$\frac{x(x+4)}{(x-1)(x+2)} = 0 \qquad x(x+4) = 0$$

$x = 0$ or $x + 4 = 0$

$x = -4$

Vertical Asymptotes: $x = -2$ and $x = 1$

$(x-1)(x+2)$ are 0

for $x = -2$ and $x = 1$

Hole(s): $\left(3, \frac{21}{10}\right)$

$$f(3) = \frac{3(3+4)}{(3-1)(3+2)} = \frac{3 \cdot 7}{2 \cdot 5} = \frac{21}{10}$$

Pr 6. Compute the difference quotient for $f(x) = \dfrac{7x}{9x-4}$.

$$f(x+h) = \frac{7(x+h)}{9(x+h)-4} = \frac{7x+7h}{9x+9h-4}$$

$$f(x+h) - f(x) = \frac{7x+7h}{9x+9h-4} - \frac{7x}{9x-4}$$

$$= \frac{(7x+7h)(9x-4)}{(9x+9h-4)(9x-4)} - \frac{(7x)(9x+9h-4)}{(9x-4)(9x+9h-4)}$$

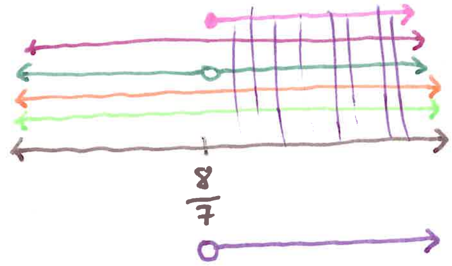
$$= \frac{63x^2 + 63xh - 28x - 28h - 63x^2 - 63xh + 28x}{(9x+9h-4)(9x-4)}$$

$$= \frac{-28h}{(9x+9h-4)(9x-4)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{-28h}{(9x+9h-4)(9x-4)}}{h}$$

$$= \frac{-28h}{(9x+9h-4)(9x-4)} \cdot \frac{1}{h} = \frac{-28}{(9x+9h-4)(9x-4)}$$

Pr 7. State the domain of $f(x) = \dfrac{(x+9)\sqrt[3]{x+5}}{(7x-8)^{\frac{11}{4}}}$, using interval notation.



Numerator:

$x+9$ is a polynomial

$\Rightarrow x \in (-\infty, \infty)$

$\sqrt[3]{x+5}$ is an odd root and is defined when $x+5$ is defined.

$x+5$ is a polynomial

$\Rightarrow x \in (-\infty, \infty)$

Denominator:

$(7x-8)^{11/4} \neq 0$ AND $(7x-8)^{11/4} = \sqrt[4]{(7x-8)^{11}}$ is an even root and is defined when

$7x-8 \neq 0$

$7x \neq 8$

$x \neq \frac{8}{7}$

$(7x-8)^{11}$ is defined AND $(7x-8)^{11} \geq 0$

$(7x-8)^{11}$ is a polynomial

$\Rightarrow x \in (-\infty, \infty)$

$7x-8 \geq 0$

$7x \geq 8$

$x \geq 8/7$

Domain: $x \in \left(\frac{8}{7}, \infty\right)$

Pr 8. Rationalize $\dfrac{\sqrt{6-x}-12}{x-21}$.

$$\left(\frac{\sqrt{6-x}-12}{x-21}\right)\cdot\left(\frac{\sqrt{6-x}+12}{\sqrt{6-x}+12}\right) = \frac{\left(\sqrt{(6-x)}\right)^2 + 12\sqrt{6-x} - 12\sqrt{6-x} - (12)^2}{(x-21)(\sqrt{6-x}+12)}$$

$$= \frac{(6-x) - 144}{(x-21)(\sqrt{6-x}+12)}$$

$$= \frac{-x-138}{(x-21)(\sqrt{6-x}+12)}$$

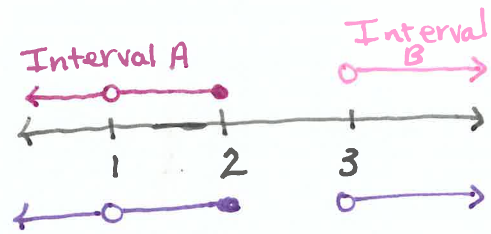
Pr 9. State the domain of $h(x) = \begin{cases} \frac{5}{x-1} & \text{if} \quad x \leq 2 \\ \sqrt{4x+1} & \text{if} \quad x > 3 \end{cases}$, using interval notation.

Rule A: $\frac{5}{x-1}$ has a domain of $x \neq 1$, so $x \leq 2$ must be broken up into

$x \in (-\infty, 1) \cup (1, 2]$

Rule B: $\sqrt{4x+1}$ is an even root and has a domain of $x \geq -1/4$, so $x > 3$ is in that interval and does not need broken up

Domain: $x \in (-\infty, 1) \cup (1, 2] \cup (3, \infty)$



Pr 10. State the domain of $f(x) = e^{\sqrt[8]{9-7x}}$, using interval notation.

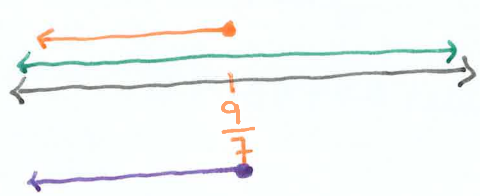
$f(x) = e^{\sqrt[8]{9-7x}}$ is an exponential function and is defined when $\sqrt[8]{9-7x}$ is defined.

$\sqrt[8]{9-7x}$ is an even root and is defined when

$9-7x$ is defined AND $9-7x \geq 0$

$9-7x$ is a polynomial $\quad -7x \geq -9$

$\Rightarrow x \in (-\infty, \infty) \quad x \leq 9/7$



Domain: $x \in (-\infty, \frac{9}{7}]$

Pr 11. Algebraically solve $16 \cdot 8^{2x-1} = 256$ for x.

$$16 \cdot 8^{2x-1} = 256$$

$$8^{2x-1} = 16$$

$$(2^3)^{3(2x-1)} = 2^4$$

$$2^{6x-3} = 2^4$$

So

$$6x-3 = 4$$

$$6x = 7$$

$$x = 7/6$$

Pr 12. You would like to save \$2500 by making an initial deposit in a savings account earning annual interest at a rate of 0.45% and leave it there for 6 years. How much should be place in the account initially, if no other deposits are made during that time and the account is compounded continuously?

$A = 2500$

$P = ?$

$r = 0.0045$

$t =$

$$A = Pe^{rt}$$

$$2500 = Pe^{0.0045(6)}$$

$$2500 = Pe^{0.027}$$

$$\frac{2500}{e^{0.027}} = P$$

Exact Value: $\$\frac{2500}{e^{0.027}}$

Rounded Value: \$2433.40