



SECTION 6.2: ANNUITIES, SINKING FUNDS, AND AMORTIZATION

Pr 1. You would like to save up \$70,000 over the next 15 years. You have \$900 that you will use as an initial deposit, and then will make quarterly payments for the next 15 years. The account has an interest rate of 7.5% per year, compounded quarterly. How much should you deposit each quarter in order to reach your goal?

$$N = 15 \cdot 4$$

$$I\% = 7.5$$

$$PV = -900$$

$$PMT = ? \longrightarrow -615.66260$$

$$FV = 70000$$

$$P/Y = 4$$

$$C/Y = 4$$

PMT: END

Quarterly payments of
\$615.66

Pr 2. Your parents started saving account, for your college expenses, when you turned three years old. They place \$100 into the account each month. If the account has an annual interest rate of 1.6%, compounded monthly.

$$N = (18-3) \cdot 12 = 180$$

$$I\% = 1.6$$

$$PV = 0$$

$$PMT = -100$$

$$FV = ? \longrightarrow 20,328.44605$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

no mention of one-time investment
deposit money

(a) How much money will be in the account when you turn 18?

There is \$20,328.45 in the account

(b) How much did your parents invest in the account?

$$\begin{aligned} \text{Amount Invested} &= N \cdot PMT \\ &= 180 \cdot 100 \\ &= \$18,000 \end{aligned}$$

Pr 3. You have deposited \$125 in you IRA at the end of each month for the past 30 years. Your investment has earned an fixed APR of 8%, compounded monthly, over the entire 30 years. Now, at age 55, you are considering retirement. If you keep the money in the account and it maintains a fixed APR of 8%, compounding monthly, what monthly payment will you receive over the next 30 years?

no mention
 $N = 30 \cdot 12$
 $I\% = 8$
 $PV = 0$
 $PMT = -125$
 $FV = ? \rightarrow 186294.9311$
 $P/Y = 12$
 $C/Y = 12$
 $PMT: END$

THEN

in the account →

empty account

$N = 30 \cdot 12$
 $I\% = 8$
 $PV = -186294.93$
 $PMT = ? \rightarrow 1366.966199$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$
 $PMT: END$

you will receive monthly payments of \$1366.97.

Pr 4. You owe \$4500 on a credit card that charges 14.5% per year, compounded monthly. If you only pay the mininum amount of \$60 per month, how long will it take you to pay off the credit card, if you make no additional purchases on the card?

(+) as you receive
 pay off / owe nothing →
 $N = t \cdot 12 \rightarrow ? \rightarrow 197.0810778$
 $I\% = 14.5$
 $PV = 4500$
 $PMT = -60$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$
 $PMT: END$

$N = 198$ (always round up)
 $12t = 198$
 $t = 16.5 \text{ years}$

16.5 years to pay off the card.

Pr 5. The Kleins have decided to buy a house. They can make a down payment of \$30,000 and monthly payments up to \$800. The lowest rate that they were quoted was 7.2% per year, compounded monthly, for 30 years.

(a) What is the most expensive house they can afford to buy?

$$N = 30 \cdot 12$$

↳ PV + Down Payment

$$I\% = 7.2$$

$$PV = ? \rightarrow 117\,857.0854$$

$$PMT = -800$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

The loan is for \$117,857.09

$$\text{House Price} = 117\,857.09 + 30\,000$$

$$= \$147,857.09 \leftarrow \text{most expensive house they can afford.}$$

(b) Suppose the house that they decide to buy has a price tag of \$139,000. What are the monthly payments that they would pay to amortize (payoff) the loan?

$$N = 30 \cdot 12$$

$$I\% = 7.2$$

$$PV = 139\,000 - 30\,000 = 109\,000$$

$$PMT = ? \rightarrow -739.8791483$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

Their monthly payments are \$739.88.

Pr 6. The Phredds is buying a some land for a price of \$224,000. They make a 15% down payment and borrow the rest from a bank at an interest rate of 3.52% per year, compounded monthly. The loan will have to be paid off in 15 years.

(a) What is their monthly mortgage payments be?

$$N = 15 \cdot 12 =$$

$$I\% = 3.52$$

$$PV = 0.85(224000) = 190400$$

$$PMT = ? \rightarrow -1363.007148$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

The monthly payment is \$1363.01

(b) What is the outstanding balance on the loan after seven year?

$$N = 7 \cdot 12 = 84 \text{ payments made}$$

$$I\% = 3.52$$

$$PV = 190400$$

$$PMT = -1363.01$$

$$FV = ? \rightarrow 113894.5143$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

OR

$$N = (15 - 7) \cdot 12 = 96 \text{ payments remaining}$$

$$I\% = 3.52$$

$$PV = ? \rightarrow 113895.02$$

$$\text{From (a)} \text{ PMT} = -1363.01$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

The outstanding balance is \$113,895

(c) After seven years of making payments, how much equity will they have in the land? Assume the value of the land remains constant.

$$\begin{aligned} \text{Equity} &= \text{Purchase Price} - \text{Outstanding Balance} \\ &= 224000 - 113895 \\ &= \$110,105 \end{aligned}$$

(d) How much interest did the Phredds pay the bank?

$$\begin{aligned} \text{Interest Paid} &= \text{Amount Paid to Bank} - \text{Loan Amount} \\ &= N \cdot \text{PMT} - PV \\ &= 180(1363.01) - 190400 \\ &= \$54,941.80 \end{aligned}$$

If 15% down then owe 85%.

From (a)

From (a)

Pr 7. You currently owe \$4,500 to the store for the furniture that you purchased. You made a down payment of \$3,000 and have been making payments of \$350 each month for the last three years. The store is charging you interest of 6.3% APR, compounded monthly, on the loan.

$$N = 3 \cdot 12$$

$$I\% = 6.3$$

$$PV = ? \rightarrow 15180.4906$$

$$PMT = -350$$

$$FV = -4500$$

$$P/Y = 12$$

$$C/Y = 12$$

$$PMT: END$$

Need amount borrowed

you borrowed \$15,180.49

you still owe

(a) What was the purchase price of the furniture?

$$\text{Purchase Price} = PV + \text{Down Payment}$$

$$= 15180.49 + 3000$$

$$= \$18,180.49$$

(b) How many more payments will you have to make until the furniture is paid off?

$$N = ? \rightarrow 49.346559$$

$$I\% = 6.3$$

$$PV = 15180.49$$

$$PMT = -350$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$PMT: END$$

$N = 36$ payments already made

$N = 50$ total payment

$50 - 36 = 14$ payments remain

paid off

$$18 = N = t \cdot m$$

Pr 8. If you buy a television set for \$2500 and agree to pay for it in eighteen equal monthly payments with an annual interest rate of 18%, compounded monthly, how much are your monthly payments?

$$N = 18$$

$$I\% = 18$$

$$PV = 2500$$

$$PMT = ? \rightarrow -159.5144544$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

your monthly payments are \$159.51

(a) How much are your monthly payments?

\$159.51

(b) How much of the first payment goes towards interest?

Periodic interest rate \cdot Outstanding Balance = Amount to Interest

$$\frac{.18}{12} \cdot (2500) = 37.5$$

\$37.50 of the first payment goes to Interest

(c) How much of the 10th payment will go towards the balance?

$$N = 9$$

$$I\% = 18$$

$$PV = ? \rightarrow 1333.586118$$

$$PMT = -159.51$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT: END

9 payments remain

$$\begin{aligned} \text{Towards} \\ \text{Balance} &= PMT - \text{Amount to Interest} \\ &= 159.51 - \left(\frac{0.18}{12}\right)(1333.59) \end{aligned}$$

$$= 159.51 - 20.00$$

$$\text{Towards} \\ \text{Balance} = \$139.51$$