Solutions

Section 1.1: Basic Matrix Operations

1. Given matrices A, B, C, D, and E below, determine each part.

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} \quad 2 \times 3$$

$$B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix} \quad 2 \times 3$$

$$C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix} \quad 3 \times 2$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix} \quad 2 \times 2$$

$$E = \begin{bmatrix} v & 81 \\ 4y & (42-w) \end{bmatrix} \quad 2 \times 2$$

a. State the dimensions of each matrix. row, column

b. If it exists, state the value of b_{23}. $b_{23} = 8$

c. If it exists, state the value of c_{43}. c_{43} does not exist, no 4th row or 3rd column

d. If $M = B^T$, state the value of m_{31}, if it exists.

$M = \begin{bmatrix} -9 & -1 \\ 0 & -5 \\ 3 & 8 \end{bmatrix}$ (3 × 2) so $m_{31} = 3$

e. Compute $A + B$, if possible. If the operation is not possible, explain why.

$A + B = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} + \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix} = \begin{bmatrix} 7+(-9) & 2+0 & 4+(3) \\ 6+(-1) & 5+(-5) & 0+8 \end{bmatrix}$

2×3 2×3 — same size, can add

$$A + B = \begin{bmatrix} -2 & 2 & 7 \\ 5 & 0 & 8 \end{bmatrix}$$

f. Compute $C^T - 4A + 0.5B$, if possible. If the operation is not possible, explain why.

$C^T - 4A + 0.5B$

$\begin{bmatrix} -2 & -y & 7 \\ w & 0 & (x+1) \end{bmatrix} + \begin{bmatrix} -28 & -8 & -16 \\ -24 & -20 & 0 \end{bmatrix} + \begin{bmatrix} -\frac{9}{2} & 0 & \frac{3}{2} \\ -\frac{1}{2} & -\frac{5}{2} & 4 \end{bmatrix} =$

2×3 2×3 2×3 — same size, can add/subtract

$$C^T - 4A + 0.5B = \begin{bmatrix} -34.5 & (-y-8) & -7.5 \\ (w-24.5) & -22.5 & (x+5) \end{bmatrix}$$

g. Compute $2E - 3D$, if possible. If the operation is not possible, explain why.

$2E - 3D$

$\begin{bmatrix} 2v & 162 \\ 8y & (84-2w) \end{bmatrix} + \begin{bmatrix} -42 & (-6x+27) \\ (-21y-36) & -45 \end{bmatrix} = \begin{bmatrix} (2v-42) & (-6x+189) \\ (-13y-36) & (-2w+39) \end{bmatrix}$

2×2 2×2 — same size, can subtract

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix}$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix} \qquad E = \begin{bmatrix} v & 81 \\ 4y & (42-w) \end{bmatrix}$$

h. Compute $(B^T + C)^T$, if possible. If the operation is not possible, explain why.

$(B^T + C)^T$

$3\times2 \quad 3\times2$

same size can add

$$B^T + C = \begin{bmatrix} -9 & -1 \\ 0 & -5 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix} = \begin{bmatrix} -11 & (w-1) \\ -y & -5 \\ 10 & (x+9) \end{bmatrix}$$

$$(B^T + C)^T = \begin{bmatrix} -11 & -y & 10 \\ (w-1) & -5 & (x+9) \end{bmatrix}$$

2×3

i. If $A + B = C^T$, determine the values of w, x, and y.

$$A + B = \begin{bmatrix} -2 & 2 & 7 \\ 5 & 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 7 \\ 5 & 0 & 8 \end{bmatrix} = \begin{bmatrix} -2 & -y & 7 \\ w & 0 & (x+1) \end{bmatrix}$$

So $-2 = -2$ ✓ $\quad 2 = -y \quad -2 = y \quad 7 = 7$ ✓

$5 = w \qquad 0 = 0 \qquad 8 = x+1 \quad 7 = x$

Thus $w = 5$, $y = -2$, $x = 7$

2. Use the given information about the sizes of matrices A, B, C, D, and E to determine the size of the resulting matrix, if the computation is possible. If the product is not possible, explain why.

A is a 1×2, B is a 1×2, C is a 2×3, D is a 2×3 E is a 3×2

a. $\frac{2}{3}B$ is a 1×2 matrix

b. $(B + C)^T$ As B and C are different sizes the sum is not possible

$1\times2 \quad 2\times3$

different sizes

c. $E^T - D + 2C$ is a 2×3 matrix

$2\times3 \quad 2\times3 \quad 2\times3$

all the same size

3. Solve the matrix equation for matrix X.

$$8X - \begin{bmatrix} 120 & 250 \\ 480 & 275 \end{bmatrix} = \begin{bmatrix} 270 & -140 \\ -20 & 225 \end{bmatrix} + 3X$$

$$5X = \begin{bmatrix} 270 & -140 \\ -20 & 225 \end{bmatrix} + \begin{bmatrix} 120 & 250 \\ 480 & 275 \end{bmatrix}$$

$$5X = \begin{bmatrix} 390 & 110 \\ 460 & 500 \end{bmatrix}$$

$$X = \frac{1}{5}\begin{bmatrix} 390 & 110 \\ 460 & 500 \end{bmatrix}$$

$$X = \begin{bmatrix} 78 & 22 \\ 92 & 100 \end{bmatrix}$$

4. Solve the matrix equation, $B + 4X = 2E - 6X$, for matrix X, assume that all matrices, B, X, and E are the same size.

$$B + 4X = 2E - 6X$$

$$10X = 2E - B$$

$$X = \frac{1}{10}(2E - B)$$

or

$$X = \frac{1}{5}E - \frac{1}{10}B$$

5. An online streaming service records the number of downloads based genre and media type. During the month of January 3000 science fiction novels, 4500 science fiction movies, 4200 thiller novels, 7000 thiller movies, 1200 romance novels, and 6800 romantic movies were downloaded, while in February the downloads were 3800, 2900, 2600, 5100, 4500, and 9700 respectively.

a. Write a 3×2 matrix, J, representing the online streaming service data in January.

$$J = \begin{matrix} \\ SF \\ T \\ R \end{matrix} \begin{matrix} \begin{matrix} N & M \end{matrix} \\ \begin{bmatrix} 3000 & 4500 \\ 4200 & 7000 \\ 1200 & 6800 \end{bmatrix} \end{matrix}$$

b. Write a matrix operation that would represent the change in sales from January to February.

$$F - J = \begin{matrix} \\ SF \\ T \\ R \end{matrix} \begin{matrix} \begin{matrix} N & M \end{matrix} \\ \begin{bmatrix} 3800 & 2900 \\ 2600 & 5100 \\ 4500 & 9700 \end{bmatrix} \end{matrix} - \begin{bmatrix} 3000 & 4500 \\ 4200 & 7000 \\ 1200 & 6800 \end{bmatrix} = \begin{matrix} \begin{matrix} N & M \end{matrix} \\ \begin{bmatrix} 800 & -1600 \\ -1600 & -1900 \\ 3300 & 2900 \end{bmatrix} \end{matrix} \begin{matrix} SF \\ T \\ R \end{matrix}$$

c. Explain in the context of the application what entry c_{12} tells us, if $C = J + F$. Assume $F = \begin{matrix} \\ SF \\ \\ \end{matrix} \begin{matrix} \begin{matrix} \\ M \end{matrix} \\ \begin{bmatrix} 3800 & 2900 \\ 2600 & 5100 \\ 4500 & 9700 \end{bmatrix} \end{matrix}$.

c_{12} is the sum of the science fiction movies downloading in January and February, $2900 + 4500 = 7400$

d. If the streaming service expects a 30% increase in all sales from February to March, how many romantic movies are expected to be downloaded?

in March

$1 + 0.3 = 1.3$ times as many

$$1.3F = 1.3 \begin{bmatrix} 3800 & 2900 \\ 2600 & 5100 \\ 4500 & 9700 \end{bmatrix} = \begin{bmatrix} — & — \\ — & — \\ — & 12610 \end{bmatrix}$$

12,610 romantic movies are expected to be downloaded in March.

Section 1.2: Matrix Multiplication

1. Use the given information about the sizes of matrices A, B, C, D, and E to determine the size of the resulting matrix, if the computation is possible. If the product is not possible, explain why.

A is a 1×3, B is a 2×2, C is a 2×4, D is a 3×3, E is a 4×3, F is a 2×2

a. BC

$(2 \times 2)(2 \times 4)$ $\Rightarrow$ BC is a 2×4 matrix

b. CB

$(2 \times 4)(2 \times 2)$

CB is not possible as the inner dimensions are not equal

c. $7CC^T$

$7C$ is a 2×4

C^T is a 4×2

$7CC^T$

$(2 \times 4)(4 \times 2)$ $\Rightarrow$ $7CC^T$ is a 2×2 matrix

d. FCE

$(2 \times 2)(2 \times 4)(4 \times 3)$ $\Rightarrow$ FCE is a 2×3 matrix

e. $(F + B)C$

$F + B$ $\Rightarrow$ $F+B$ is a 2×2 matrix

2×2 2×2 same size

then

$(F+B)C$ $\Rightarrow$ $(F+B)C$ is a 2×4 matrix

$(2 \times 2)(2 \times 4)$

f. $C^T(2B - 3F)$

$(2B - 3F)$ is a 2×2 matrix

C^T is a 4×2 matrix

$C^T(2B-3F)$ $\Rightarrow$ $C^T(2B-3F)$ is a 4×2 matrix

$(4 \times 2)(2 \times 2)$

2. Given matrices A, B, C, D, and E below, determine each matrixproduct, if possible.

$$A = \begin{bmatrix} 7 & 2 & 4 \end{bmatrix} \quad 1\times 3$$

$$B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix} \quad 2\times 3$$

$$C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix} \quad 3\times 2$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix} \quad 2\times 2$$

$$E = \begin{bmatrix} v & 81 \\ 4 & (42-w) \end{bmatrix} \quad 2\times 2$$

a. AD

$(1\times 3)(2\times 2)$ $\neq$ AD is not possible

b. $-2BA^T$

$(2\times 3)(3\times 1)$ $=$ ✓

$$-2B = \begin{bmatrix} 18 & 0 & -6 \\ 2 & 10 & -16 \end{bmatrix} \qquad A^T = \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix}$$

$$-2BA^T = \begin{bmatrix} 18 & 0 & -6 \\ 2 & 10 & -16 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 18(7)+0(2)+(-6)(4) \\ 2(7)+10(2)+(-16)(4) \end{bmatrix}$$

2×1

$$-2BA^T = \begin{bmatrix} 102 \\ -30 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix} \quad 2\times3 \qquad C = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix} \quad 3\times2$$

$$D = \begin{bmatrix} 14 & (2x-9) \\ (7y+12) & 15 \end{bmatrix} \quad 2\times2 \qquad E = \begin{bmatrix} v & 81 \\ 4 & (42-w) \end{bmatrix} \quad 2\times2$$

c. CC^T

$(3\times2)(2\times3)$ = ✓

$$C^T = \begin{bmatrix} -2 & -y & 7 \\ w & 0 & (x+1) \end{bmatrix}$$

$$CC^T = \begin{bmatrix} -2 & w \\ -y & 0 \\ 7 & (x+1) \end{bmatrix} \begin{bmatrix} -2 & -y & 7 \\ w & 0 & (x+1) \end{bmatrix} = \begin{bmatrix} (-2)(-2)+w(w) & (-2)(-y)+w(0) & (-2)7+w(x+1) \\ (-y)(-2)+0(w) & (-y)(-y)+0(0) & (-y)7+0(x+1) \\ 7(-2)+(x+1)w & 7(-y)+(x+1)(0) & 7(7)+(x+1)(w) \end{bmatrix}$$

3×3

$$CC^T = \begin{bmatrix} (4+w^2) & 2y & (-14+wx+w) \\ 2y & y^2 & -7y \\ (-14+wx+w) & -7y & (49+(x+1)^2) \end{bmatrix}$$

d. $(3D+4E)B$

$3D + 4E$

$2\times2 \quad 2\times2$

same size can add

$$3D + 4E = \begin{bmatrix} 42 & (6x-27) \\ (21y+36) & 45 \end{bmatrix} + \begin{bmatrix} 4v & 324 \\ 16 & (168-4w) \end{bmatrix}$$

$$3D+4E = \begin{bmatrix} (42+4v) & (6x+297) \\ (21y+52) & (-4w+213) \end{bmatrix} \quad 2\times2$$

$(3D+4E)B$

$(2\times2)(2\times3)$ = ✓

$$(3D+4E)B = \begin{bmatrix} (42+4v) & (6x+297) \\ (21y+52) & (-4w+213) \end{bmatrix} \begin{bmatrix} -9 & 0 & 3 \\ -1 & -5 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (42+4v)(-9)+(6x+297)(-1) & (42+4v)(0)+(6x+297)(-5) & (42+4v)(3)+(6x+297)(8) \\ (21y+52)(-9)+(-4w+213)(-1) & (21y+52)(0)+(-4w+213)(-5) & (21y+52)(3)+(-4w+213)(8) \end{bmatrix}$$

2×3

$$(3D+4E)B = \begin{bmatrix} (-36v-6x-675) & (-30x-1485) & (12v+48x+2502) \\ (189y+4w-681) & (20w-1065) & (63y-32w+1860) \end{bmatrix}$$