Solution

SECTION 2.3: SYSTEMS OF TWO EQUATIONS IN TWO UNKNOWNS

1. State the type of linear system given without graphing or actually computing the solution. Then, state the number of solutions.

$$\begin{array}{l} L_1 \\ L_2 \end{array}\begin{cases} 4x + 5y = 12 \\ 2x = -\frac{5}{2}y + 6 \end{cases}$$

L_1: $4x + 5y = 12$

$5y = -4x + 12$

$y_1 = -\frac{4}{5}x + \frac{12}{5}$

$m_1 = -\frac{4}{5}$

$b_1 = \frac{12}{5}$

L_2: $2x = -\frac{5}{2}y + 6$

$-\frac{5}{2}y = 2x - 6$

$y_2 = -\frac{4}{5}x + \frac{12}{5}$

$m_2 = -\frac{4}{5}$

$b_2 = \frac{12}{5}$

As $m_1 = m_2$ AND $b_1 = b_2$, the system is dependant and has infinitely many solutions.

2. Find the value of k so that the following system of equations has no solution.

$m_1 = m_2$ BUT $b_1 \neq b_2$

$$\begin{array}{l} L_1 \\ L_2 \end{array}\begin{cases} y = \frac{5}{4}x + 1 \\ 10x - ky = -6 \end{cases}$$

L_1: $y = \frac{5}{4}x + 1$

$m_1 = \frac{5}{4}$

$b_1 = 1$

L_2 $10x - ky = -6$

$-ky = -10x - 6$

$y = \frac{10}{k}x + \frac{6}{k}$

$m_2 = \frac{10}{k}$

$b_2 = \frac{6}{k}$

$M_1 = m_2$

$\frac{5}{4} = \frac{10}{k}$

$5k = 40$

$k = 8$

Check

$b_1 \neq b_2$, when $k = 8$

$1 \neq \frac{6}{8}$

$1 \neq \frac{6}{8}$ True

So when $k = 8$ the system has no solution.

3. Solve each system using the stated method. Write any solutions as ordered pairs with exact values. For parametric solutions use p as your parameter.

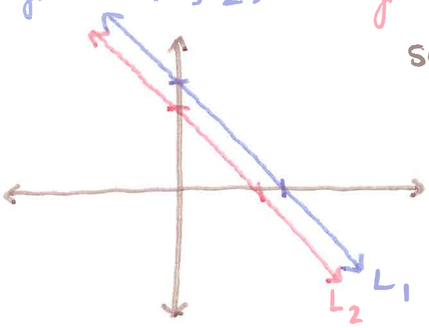
(a) $\begin{cases} 3x + 2y = 5 \\ y = -\frac{3}{2}x + 2 \end{cases}$, using the graphical method.

L_1: $3x+2y=5$
x-int: $(\frac{5}{3}, 0)$
y-int: $(0, \frac{5}{2})$

L_2: $y = -\frac{3}{2}x + 2$
x-int: $(\frac{4}{3}, 0)$
y-int: $(0, 2)$

same slopes

No Solution



(b) $\begin{cases} 3x - 2y = -3 \\ 5x - y = 2 \end{cases}$, using the substitution method.

$3x-2y=-3$
$5x-y=2 \Rightarrow y = 5x-2$

$3x-2(5x-2) = -3$
$3x-10x+4 = -3$
$-7x=-7$
$x=1$

$y=5(1)-2$
$y=3$

$(x,y) = (1, 3)$

(c) $\begin{cases} 3x - 2y = -4 \\ 4y = 6x + 8 \end{cases}$, using the addition method.

$(3x-2y=-4)\cdot 2 \Rightarrow 6x-4y=-8$
$4y=6x+8 \Rightarrow -6x+4y=8$

$0x+0y=0$
$0=0$ True $\Rightarrow$ Infinitely many Solutions

Let $y=p$, where p is any real #

then

$3x-2y=-4 \Rightarrow 3x-2p=-4$
$3x=2p-4$
$x=\frac{2}{3}p-\frac{4}{3}$

So $(x,y) = (\frac{2}{3}p-\frac{4}{3}, p)$

4. The production cost for a record company are \$18 per record and if they produce 60 records, then the total costs are \$1652. The company sells each record for \$40. Determine and interpret the break-even point for the record company on the production and sale of these records.

Cost: $C(x) = 18x + F$

and $(60, 1652)$

$1652 = 18(60) + F$

$572 = F$

Cost: $C(x) = 18x + 572$

Revenue: $R(x) = 40x$

So $R(x) = C(x)$

$40x = 18x + 572$

$22x = 572$

$x = 26$

$R(26) = 40(26) = 1040$

The company will cover ~~their~~ all their costs of \$1040 when 26 records are sold. The company will earn a ~~profit~~ gain when 27 or more records are produced and sold.

Break-even point: $(x, R) = (26, 1040)$

Note: $P(x) = 22x - 572$

profit = 0 ⟶ $0 = 22x - 572$

$x = 26$

5. Consumers will buy 10,000 items at a price of \$120 per item. If the price goes up by \$30 per item, then they will only buy 7600 items. Producers will not market this item below \$40, but if the price per item increases by \$15, the producers will provide 6000 items to the market. Determine and interpret the market equilibrium point for these items.

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Demand: $(10000, 120)$

$(7600, 120 + 30)$

$m = \frac{150 - 120}{7600 - 10000} = \frac{30}{-2400} = -\frac{1}{80}$

$D(x) = -\frac{1}{80}(x - 10000) + 120 = -\frac{1}{80}x + 245$

Supply: $(0, 40)$

$(6000, 40 + 25)$

$m = \frac{65 - 40}{6000 - 0} = \frac{25}{6000} = \frac{1}{240}$

$S(x) = \frac{1}{240}(x - 0) + 40 = \frac{1}{240}x + 40$

Equilibrium Pt: $D(x) = S(x)$ (x, p)

$-\frac{1}{80}x + 245 = \frac{1}{240}x + 40$

$205 = \frac{1}{60}x$

$12300 = x$

$p(12300) = -\frac{1}{80}(12300) + 245$

$p = 91.25$

The equilibrium point is $(12300, 91.25)$.

If producers make 12,300 item and sell them at \$91.25 each, consumers will purchase all 12,300 items.

1. Set up, but do not solve, a system of linear equations which could be used to solve the problem.

You have \$50,000 to invest in Fund A and Fund B. Fund A pays 7.4% and Fund B pays 9.8%. How much do you invest in each to get a return of \$4,072 per year?

$a :=$ the amount, in dollars, invested in Fund A
$b :=$ the amount, in dollars, invested in Fund B

$a + b = 50000$ (Total invested)
$0.074a + 0.098b = 4072$ (Returns)

2. Write the corresponding augmented matrix for the systems of linear equations.

$$\begin{cases} 4x + 2y + 3z = 72 \\ 2y - 3z = 12 \\ -x + 9 = 5y + z \end{cases}$$

$\Rightarrow +x + 5y + z = 9$

$$\begin{array}{cccc} x & y & z & \text{constants} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 4 & 2 & 3 & 72 \\ 0 & 2 & -3 & 12 \\ 1 & 5 & 1 & 9 \end{array}\right]$$

3. Write a system of linear equations which corresponds to the augmented matrix. Assume the variables are x and y or x, y, and z.

$$\begin{array}{c|cccc} & x & y & z & \text{constants} \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \end{array}$$
$$\left[\begin{array}{ccc|c} -2 & -6 & -10 & -12 \\ 0 & 1 & 2 & 3 \\ 2 & 1 & 2 & -5 \end{array}\right]$$

$-2x - 6y - 10z = -12$
$y + 2z = 3$
$2x + y + 2z = -5$

4. Perform the indicated row operation and write the resulting matrix.

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 5 & 2 & 19 \end{array}\right] \xrightarrow{-5R_1 + R_2 \to R_2}$$

$$\begin{array}{rrrrr} & -5R_1 = & -5 & 20 & -5 \\ + & R_2 = & 5 & 2 & 19 \\ \hline & & 0 & 22 & 14 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 22 & 14 \end{array}\right]$$

5. State if the matrix is in reduced row-echelon form. If the matrix is not in reduced row-echelon form, state which of the four conditions is first violated, as stated in the definition.

(a) $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & 1 & 0 \end{array}\right]$

Not in rref

The column with a leading 1 should have 0's as all other entries in the column.

(b) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

In rref

6. Solve each system of linear equations using matrices. Write your answer as an ordered pair or ordered triple, as appropriate. For parametric solutions use t as your parameter.

(a) $\begin{cases} 3x + 5y = -2 \\ -9x - 15y = 6 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & 5 & -2 \\ -9 & -15 & 6 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 5/3 & -2/3 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{c} x + \frac{5}{3}y = -\frac{2}{3} \\ 0 = 0 \end{array}$$

Let $y = t$, where t is any real number

then $x + \frac{5}{3}y = -\frac{2}{3}$

$x + \frac{5}{3}t = -\frac{2}{3}$

$x = -\frac{5}{3}t - \frac{2}{3}$

Solution: $(x, y) = \left(-\frac{5}{3}t - \frac{2}{3}, t\right)$

(b) $\begin{cases} x + 4z = 0 \\ x + y = -2z + 1 \\ 6z - 3x = -3y + 15 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 1 & 1 & 2 & 1 \\ -3 & 3 & 6 & 15 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \quad \begin{array}{l} x = -2 \\ y = 2 \\ z = \frac{1}{2} \end{array}$$

Solution: $(x, y, z) = \left(-2, 2, \frac{1}{2}\right)$