EXAM 1 REVIEW OVER CHAPTERS 1 AND 2

- Basic Matrix Operations
- Matrix Multiplication
- Review of Lines
- Modeling with Linear Functions
- Systems of Two Equations in Two Unknowns
- Setting Up and Solving Systems of Linear Equations

1. State the dimensions of matrix A and then state the value of a_{32} given

$$A = \begin{array}{c c} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} -6 & 7x & y \\ 5w & -2 & 9 \\ -3y & 1 & 0 \\ 4x & 8 & 10w \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}.$$

A is a 4 x 3 matrix

$a_{32} = 1$

row 3 column 2

2. If A is a 2×1 matrix, B is a 2×1 matrix, and C is a 1×2 matrix, determine the size of $(3A + 4B)^T - 0.5C$, if possible.

3A is 2x1
4B is 2x1
3A + 4B is a 2x1
same size
$(3A+4B)^T$ is a 1x2
0.5C is a 1x2

so

$(3A+4B)^T - 0.5C$ is a 1x2
same size

3. Determine the value of w, x, and y given $\begin{bmatrix} 2 & w-1 \\ 3 & 4x \end{bmatrix} - \begin{bmatrix} y & -6 \\ -8 & 12 \end{bmatrix}^T = 3\begin{bmatrix} -1 & 7 \\ 3 & -4 \end{bmatrix}$

$$\begin{bmatrix} 2 & (w-1) \\ 3 & 4x \end{bmatrix} - \begin{bmatrix} y & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} -3 & 21 \\ 9 & -12 \end{bmatrix}$$

$$\begin{bmatrix} (2-y) & (w-1)+8 \\ 9 & (4x-12) \end{bmatrix} = \begin{bmatrix} -3 & 21 \\ 9 & -12 \end{bmatrix} \Rightarrow$$

$2 - y = -3$
$-y = -5$
$y = 5$

$w + 7 = 21$
$w = 14$

$9 = 9$ ✓

$4x - 12 = -12$
$4x = 0$
$x = 0$

$\boxed{w = 14, \quad x = 0, \quad y = 5}$

4. If A is a 2×1 matrix, B is a 2×1 matrix, and C is a 3×2 matrix, determine the size of CAB^T, if possible.

CA is a 3×1

$\underbrace{(3\times 2)(2\times 1)}_{= \checkmark}$

B^T is a 1×2

CAB^T is a 3×2

$\underbrace{(3\times 1)(1\times 2)}_{= \checkmark}$

5. Compute $\begin{bmatrix} -3 & 4x & 2 \\ 5w & 0 & 4y \end{bmatrix} \begin{bmatrix} -6 & 4m \\ 2n & 3 \\ -p & 0 \end{bmatrix}$. $\underbrace{(2\times 3)(3\times 2)}_{= \checkmark} = 2 \times 2$

$$\begin{bmatrix} -3 & 4x & 2 \\ 5w & 0 & 4y \end{bmatrix} \begin{bmatrix} -6 & 4m \\ 2n & 3 \\ -p & 0 \end{bmatrix} = \begin{bmatrix} (-3)(-6)+(4x)(2n)+2(-p) & (-3)(4m)+(4x)(3)+2(0) \\ (5w)(-6)+0(2n)+(4y)(-p) & (5w)(4m)+0(3)+(4y)(0) \end{bmatrix}$$

$$= \begin{bmatrix} (18+8xn-2p) & (-12m+12x) \\ (-30w-4yp) & 20wm \end{bmatrix}$$

6. There are three convenience stores in Riley. Last week, the east store sold 88 gallons of milk, 48 bags of potato chips, 16 boxes of devil food cakes, and 112 cans of soda. The west store sold 105 bags of potato chips, 72 gallons of milk, 21 boxes of devil food cakes, and 147 cans of soda. The north store sold 60 boxes of devil food cakes, 40 bags of potato chips, 50 cans of soda, but no gallons of milk. If all three stores sell a gallon of milk for \$1.59, a can of soda for \$0.79, a bag of potato chips for \$1.19 and a box of devil food cakes for \$1.99, use matrix multiplication to compute how much money did each store bring in last week?

\$ 1 × 3 stores result

$$\begin{array}{c} \\ \$ \end{array}\begin{array}{c} \begin{array}{cccc} M & S & C & DF \end{array} \\ \begin{bmatrix} 1.59 & 0.79 & 1.19 & 1.99 \end{bmatrix} \end{array} \begin{array}{c} \\ M \\ S \\ C \\ DF \end{array} \begin{array}{c} \begin{array}{ccc} \text{East} & \text{West} & \text{North} \end{array} \\ \begin{bmatrix} 88 & 72 & 0 \\ 112 & 147 & 50 \\ 48 & 105 & 40 \\ 16 & 21 & 60 \end{bmatrix} \end{array} = \begin{array}{c} \\ \$ \end{array}\begin{array}{c} \begin{array}{ccc} \text{East} & \text{West} & \text{North} \end{array} \\ \begin{bmatrix} 317.36 & 397.35 & 206.5 \end{bmatrix} \end{array}$$

The east store brought in \$317.36, while the west store brought in \$397.35, and the north store brought in \$206.50.

7. Write the equation of the line that passes through the point $(6, -7)$ and has a slope of zero.

$\underbrace{\text{slope of zero}}_{m=0}$ $\Rightarrow$ horizontal line

$y = -7$

8. You have a line which passes through the points $(-3, -4)$ and $\left(\frac{1}{2}, \frac{2}{3}\right)$. If x decreases by 8 units, what is the corresponding change in y? Δy

$\Delta x = -8$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - \frac{2}{3}}{-3 - \frac{1}{2}} = \frac{4}{3}$

$m = \frac{\Delta y}{\Delta x}$

$\frac{4}{3} = \frac{\Delta y}{-8}$

$-\frac{32}{8} = \Delta y$

If x decreases by 8 units, y decreases by $\frac{32}{8}$ units.

9. An automobile purchased for use by the manager of a firm at a price of \$14,000 is to be depreciated usinga linear model over ten years. What will the book value of the automobile be at the end of five years if the automobile has a scrap value of \$1,000 at the end of 10 years?

$t = 5$

$(0, 14000)$ $(10, 1000)$

$m = \frac{1000 - 14000}{10 - 0} = -1300$

$V(t) = -1300t + 14000$

$V(5) = -1300(5) + 14000$

$V(5) = 7500$

The book value is \$7,500 after 5 yrs.

10. Dave sells widgets at his widget stand. He buys the widgets for \$5 each. When he sells 30 in a month, then his profit is \$276. When he sells 20 widgets in a month, then his cost for that month is \$514.

(a) Determine the linear cost function.

$m = 5$

$(20, 514)$

$C(x) = 5x + F$

$514 = 5(20) + F$

$514 = 100 + F$

$414 = F$

$\boxed{C(x) = 5x + 414}$

(b) Determine the linear revenue function.

$R(x) = p \cdot x$

$R(x) = 28x$

(c) Determine the linear profit function.

$(30, 276)$

$P(x) = R(x) - C(x)$

$P(x) = px - [5x + 414]$

$P(30) = p \cdot (30) - [5(30) + 414]$

$276 = 30p - 564$

$840 = 30p$

$28 = p$

$\boxed{P(x) = 23x - 414}$

(d) Determine and interpret the break-even point.

$R(x) = C(x)$

$28x = 5x + 414$

$23x = 414$

$x = 18$

$R(18) = 28(18)$

$= 504$

$P(x) = 0$

$23x - 414 = 0$

$23x = 414$

$x = 18$

$(18, 504)$

If Dave buys and sells 18 widgets bringing in a revenue of \$504 to cover all his costs and have a profit of zero.

consumers

11. If an ipod costs \$400, 2000 sell. If the price increases to \$500, then 1500 sell. The producer is willing to provide 700 ipods if the price is \$580 and are willing to provide 1300 ipods when the price is \$940. Assume supply and demand are linear.

(a) Determine the linear supply equation.

(700, 580)
(1300, 940)

$$m = \frac{940-580}{1300-700} = \frac{3}{5}$$

$$p(x) = \frac{3}{5}(x-700)+580$$
$$p(x) = \frac{3}{5}x + 160$$

$S(x) = p(x) = \frac{3}{5}x + 160$

(b) Determine the linear demand equation.

(2000, 400)
(1500, 500)

$$m = \frac{400-500}{2000-1500} = \frac{-1}{5}$$

$$p(x) = -\frac{1}{5}(x-2000)+400$$
$$p(x) = -\frac{1}{5}x + 800$$

$D(x) = p(x) = -\frac{1}{5}x + 800$

(c) Determine and interpret the equilibrium point.

$$S(x) = D(x)$$
$$\frac{3}{5}x + 160 = -\frac{1}{5}x + 800$$
$$\frac{4}{5}x = 640$$
$$x = 800$$
$$p(800) = \frac{3}{5}(800) + 160 = 640$$

(800, 640)

If producers supply 800 ipods at \$640 each, consumers will purchase all 800 ipods.

Extra

12. Determine the value of k so that the following system of linear equations has exactly one solution.

$$-2x + ky = 24$$
$$3x - 8y = 35$$

$m_1 \neq m_2$

$L_1: -2x + ky = 24$
$ky = 2x + 24$
$y = \frac{2}{k}x + \frac{24}{k}$
$m_1 = \frac{2}{k}$

$L_2: 3x - 8y = 35$
$-8y = -3x + 35$
$y = \frac{3}{8}x + \frac{-35}{8}$
$m_2 = \frac{3}{8}$

$$\frac{2}{k} \neq \frac{3}{8}$$
$$16 \neq 3k$$
$$\frac{16}{3} \neq k$$

The system has exactly one solution as long as $k \neq \frac{16}{3}$

Extra **13.** Set up and solve the following problem as a system of linear equations.

Link has \$17,360 to invest. He decides to invest in three different companies. The QX company costs \$130 per share and pays dividends of \$1.50 per share each year. The RY company costs \$75 per share and pay dividends of \$1.00 per share each year. The KZ company costs \$90 per share and pays \$2.00 per share per year in dividends. Link wants to have twice as much money in the RY company as in the KZ company. Link also wants to earn \$252 in dividends per year. How much should Link invest in each company to meet his goals?

$x :=$ the number of shares of QX company purchased

$y :=$ the number of shares of RY company purchased

$z :=$ the number of shares of KZ company purchased

$$\left[\begin{array}{ccc|c} 130 & 75 & 90 & 17360 \\ 1.5 & 1 & 2 & 252 \\ 0 & 1 & -2 & 0 \end{array}\right] \xrightarrow{\text{rref}}$$

$130x + 75y + 90z = 17360$ (\$ invested)

$1.5x + 1y + 2z = 252$ (dividends)

$y = 2z$ (ratio)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 56 \\ 0 & 1 & 0 & 84 \\ 0 & 0 & 1 & 42 \end{array}\right] \quad \begin{array}{l} x = 56 \\ y = 84 \\ z = 42 \end{array}$$

Link should invest \$7280 in QX, \$6300 in RY, and \$3780 in KZ.

Extra **14.** Write the augmented matrix corresponding to the given system of linear equations.

$$\begin{array}{l} 1 \quad 2x - 5y = 4 \\ 2 \quad -4x + 2y - 7z = -5 \\ \quad -y + 4 = 3z \Rightarrow -y - 3z = -4 \end{array}$$

$$\begin{array}{c} 1 \\ 2 \\ \end{array} \left[\begin{array}{ccc|c} 2 & -5 & 0 & 4 \\ -4 & 2 & -7 & -5 \\ 0 & -1 & -3 & -4 \end{array}\right]$$

Extra **15.** Determine if the augmented matrix is in reduced row-echelon form or not.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

In rref

Extra **16.** Solve the system of linear equations, using technology.

$$y = x - 3 \quad \Rightarrow \quad 3 = x - y \text{ or } x - y = 3$$
$$y - z = 1$$
$$x + z = 4$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] \qquad \begin{array}{l} x = 4 \\ y = 1 \\ z = 0 \end{array}$$

Solution: $(x, y, z) = (4, 1, 0)$

Extra **17.** Solve the system of linear equations, using technology.

$$4x + 4y - 4z = 24$$
$$2x + z = -9$$
$$-x - y + z = -6$$

$$\left[\begin{array}{ccc|c} 4 & 4 & -4 & 24 \\ 2 & 0 & 1 & -9 \\ -1 & -1 & 1 & -6 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & -9/2 \\ 0 & 1 & -3/2 & 21/2 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x + \frac{1}{2}z = -\frac{9}{2} \\ y - \frac{3}{2}z = \frac{21}{2} \\ 0 = 0 \end{array}$$

Let $z = t$, where t is any real number

$x + \frac{1}{2}z = -\frac{9}{2}$ $y - \frac{3}{2}z = \frac{21}{2}$

$x + \frac{1}{2}t = -\frac{9}{2}$ $y - \frac{3}{2}t = \frac{21}{2}$

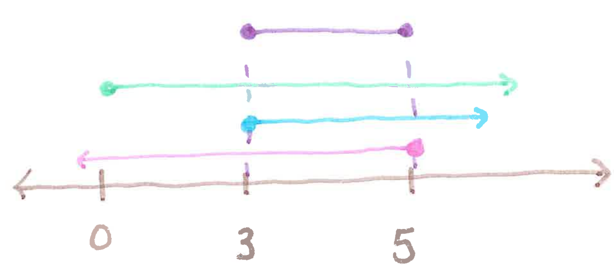
$x = -\frac{1}{2}t - \frac{9}{2}$ $y = \frac{3}{2}t + \frac{21}{2}$

Solution: $(x, y, z) = \left(-\frac{1}{2}t - \frac{9}{2}, \frac{3}{2}t + \frac{21}{2}, t\right)$ where t is any real number

Extra **18.** Assume your solution to a real-world application problem was $(x, y, z) = (10 - 2t, -3 + t, t)$. If x, y, and z represent the number of whole items produced, how many solutions does the problem actually have?

$x \geq 0, y \geq 0, z \geq 0$

$10 - 2t \geq 0$	$-3 + t \geq 0$	$t \geq 0$
$-2t \geq -10$	$t \geq 3$	
$t \leq 5$		



$3 \leq t \leq 5$ whole items $\Rightarrow$ $t = 3, 4, 5$

3 solutions, when $t = 3, 4,$ or 5