1. Determine if each of the following linear programming problems is a standard maximization problem, if it is then write the initial tableau for the linear programming problem.
   (a) Maximize: \( P = 9x + 7y + 11z \)
   Subject to: 
   - \(-2x + 4y - 6z \leq 1200\)
   - \(8x + y + 3z \leq 800\)
   - \(5x - 10y \geq -10\)
   \(x \geq 0, y \geq 0, z \geq 0\)
   A standard maximization problem
   
   (b) Maximize: \( P = 2x + 8y \)
   Subject to: 
   - \(3x - 10 \leq 2y\)
   - \(-4x + 6y \geq 8\)
   \(x \geq 0, y \geq 0\)
   Not a standard maximization problem

2. State the pivot column, pivot row, and pivot element for the given simplex tableaus below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(P) constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   (a) Pivot is 6 in \(R_2, C_1\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s_3)</th>
<th>(P) constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>3/4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   (b) Pivot is 3 in \(R_2, C_3\)
3. State the value of each variable and whether the variable is basic or non-basic for the given simplex tableaus below. Then determine if the given simplex tableau is a final tableau.

\[
\begin{array}{cccc|c}
 x & y & s_1 & s_2 & P \\
\hline
0 & -3 & 1 & 1 & 0 & 18 \\
1 & 5 & \frac{1}{3} & 0 & 0 & 50 \\
0 & 1 & \frac{1}{2} & 0 & 1 & 275 \\
\end{array}
\]

(a) Basic: \(x = 50\), \(S_2 = 18\), \(P = 275\)

Non-Basics: \(y = 0\), \(s_1 = 0\)

\[
\begin{array}{cccc|c}
 x & y & s_1 & s_2 & s_3 & P \\
\hline
1 & 0 & 0 & \frac{2}{5} & 0 & \frac{-5}{2} & 0 & 22 \\
0 & 6 & 0 & 3 & 1 & 2 & 0 & 0 \\
0 & \frac{3}{4} & 1 & 7 & 0 & 0 & 0 & 12 \\
0 & -2 & 0 & -9 & 0 & -4 & 1 & 340 \\
\end{array}
\]

(b) Basic: \(x = 22\), \(z = 12\), \(S_2 = 0\), \(P = 340\)

Non-Basic: \(y = 0\), \(s_1 = 0\), \(s_3 = 0\)

Final Tableau

4. Solve the linear programming problem, using the Simplex Method, if possible.

Maximize: \(P = 9x + 7y + 11z\)

Subject to:
- \(-2x + 4y - 6z \leq 1200\)
- \(8x + y + 3z \leq 800\)
- \(5x - 10y \geq -10\)
- \(x \geq 0, y \geq 0, z \geq 0\)

Solution: \((x, y, z) = (0, 1, \frac{799}{3})\)

\[
\begin{array}{cccc|cc}
 x & y & z & s_1 & s_2 & P \\
\hline
14 & 6 & 0 & 1 & 2 & 0 & 0 & 2800 \\
8/3 & 1/3 & 10 & 0 & 0 & 0 & 800/3 \\
-5 & 10 & 0 & 0 & 0 & 1 & 0 & 10 \\
\end{array}
\]

\[
\begin{array}{cccc|cc}
 x & y & z & s_1 & s_2 & P \\
\hline
17 & 0 & 0 & 1 & 2 & -3/5 & 0 & 2794 \\
17/10 & 0 & 10 & 0 & 0 & 0 & 799/3 \\
-1/2 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 8810/3 \\
\end{array}
\]

Solution: \((x, y, z, P) = (0, 1, \frac{799}{3}, 8810/3)\)
5. Use the Simplex Method, if possible, to solve the linear programming problem.

You have $12,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than $9,000. If Stock A earn 3% annual interest, and Stock B earn 4% annual interest, how much money should you invest in each to maximize your annual interest?

\[ a = \text{the amount, in dollars, invested in Stock A} \]
\[ b = \text{the amount, in dollars, invested in Stock B} \]
\[ I = \text{the interest earned, in dollars} \]

Maximize: \[ I = 0.03a + 0.04b \]

Subject to:
\[ a + b \leq 12000 \]
\[ a \geq 2b \]
\[ a \leq 9000 \]

\[ -a + 2b + s_2 = 0 \]
\[ a + s_3 = 9000 \]
\[ -0.03a - 0.04b + I = 0 \]

The maximum interest you can earn is $400 when you invest $8000 in Stock A and $400 in Stock B.
Section 4.1: Mathematical Experiments

1. State the sample space for each experiment:
   (a) Selecting a letter at random from the word “skate” and noting the letter.
   \[ S = \{ s, k, a, t, e \} \]

   (b) A standard 30-sided die is rolled and it is noted whether the number is a multiple of 4 or is not a multiple of 4.
   \[ S = \{ \text{multiple of 4, not a multiple of 4} \} \]

   (c) A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.
   \[ S = \{ (B,1), (B,2), (B,3), (B,4), (R,1), (R,2), (R,3), (R,4) \} \]

2. Consider the experiment of selecting a letter at random from the word “skate” and noting the letter.
   (a) State the certain event for the experiment.
   \[ C = \{ s, k, a, t, e \} \]

   (b) Given an example of an impossible event for the experiment.
   \[ I = \{ v \} \quad \text{The event “a ‘v’ is drawn”} \]

   (c) Write the outcomes in the event, \( J := \) the event “a consonant is drawn.”
   \[ J = \{ s, k, t \} \]
3. A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.

(a) State all the simple events for the experiment.

\[ \begin{align*}
\{ (B,1) \}, & \quad \{ (R,1) \}, \\
\{ (B,2) \}, & \quad \{ (R,2) \}, \\
\{ (B,3) \}, & \quad \{ (R,3) \}, \\
\{ (B,4) \}, & \quad \{ (R,4) \},
\end{align*} \]

(b) State the total number of possible events.

\[ n = 8 \]

\[ 2^8 \text{ possible events} \]

(c) Write the outcomes in the event, \( M := \text{the event \"a black or a number less an 3 is rolled.\"} \)

\[ M = \{ (B,1), (B,2), (B,3), (B,4), (R,1), (R,2) \} \]

4. An experiment consists of rolling a five-sided, noting the number showing uppermost and then spinning a spinner with five equal regions (red, blue, purple, maroon, and green), noting the color.

Let

- \( V := \text{the event \"a number greater than 3 is rolled\"} \)
- \( W := \text{the event \"an even is rolled\"} \)
- \( X := \text{the event \"the spinner lands on blue\"} \)
- \( Y := \text{the event \"the spinner lands on a color other than green\"} \)
- \( Z := \text{the event \"the spinner lands on purple or maroon.\"} \)

(a) Write the symbolic notation for the event, \( H \), that \"a number less than or equal to 3 is rolled or the spinner lands on a color other than green, but not blue.\"

\[ H = V^c \cup Y \cap X^c \]

(b) Describe the event \( \overline{Z} \cup Y^c \cap W^c \)

The event \"the spinner lands on purple or maroon or a color other than green or an odd is rolled on the die.\"

(c) Are event \( V \) and event \( W \) mutually exclusive? Explain why or why not.

\[ V = \{ (4,r), (4,b), (4,p), (4,m), (4,g), (5,r), (5,b), (5,p), (5,m), (5,g) \} \]

\[ W = \{ (2,r), (2,b), (2,p), (2,m), (2,g), (4,r), (4,b), (4,p), (4,m), (4,g) \} \]

No, because they have overlap.

\[ V \cap W = \{ (4,r), (4,b), (4,p), (4,m), (4,g) \} \]
5. Let $A$ and $B$ be two events of the sample space, $S$.

Use a two-circle Venn diagram to illustrate which region(s) contain the outcomes of the resulting events.

a. $B^c \cap A$

\[ A = \{w, x, z\} \]
\[ B = \{x, y, z\} \Rightarrow B^c = \{w, y\} \]
\[ B^c \cap A = \{w, y\} \]
\[ B^c \cap A = \{w\} \]

b. $(A \cup B) \cap A^c$

\[ A = \{w, x, z\} \Rightarrow A^c = \{y\} \]
\[ B = \{x, y, z\} \]
\[ (A \cup B) = \{w, x, y, z\} \]
\[ A \cup B = \{w, x, y, z\} \]
\[ (A \cup B) \cap A^c = \{x, y, z\} \]
\[ (A \cup B) \cap A^c = \{y\} \]