SECTION 3.4: SIMPLEX METHOD

1. Determine if each of the following linear programming problems is a standard maximization problem, if it is then write the initial tableau for the linear programming problem.

(a) Maximize: $P = 9x + 7y + 11z$

Subject to: $-2x + 4y - 6z \leq 1200$ ✓

$8x + y + 3z \leq 800$ ✓

$5x - 10y \geq -10$ $\longrightarrow$ $-5x + 10y \leq 10$ ✓

$x \geq 0,\ y \geq 0,\ z \geq 0$ ✓

① Maximization
② Constraints
$ax_1 + \ldots \leq V,\ V \geq 0$
③ Non-negativity

A standard maximization problem

(b) Maximize: $P = 2x + 8y$

Subject to: $3x - 10 \leq 2y$ $\quad 3x - 2y \leq 10$ ✓

$-4x + 6y \geq 8$ $\quad 4x - 6y \leq -8$ ✗

$x \geq 0,\ y \geq 0$ ✓

① Maximization
② Constraints
$ax_1 + \ldots \leq V,\ V \geq 0$
③ Non-negativity

Not a standard maximization problem

2. State the pivot column, pivot row, and pivot element for the given simplex tableaus below.

(a)
$$\begin{array}{c}
\begin{array}{cccccc} x & y & s_1 & s_2 & P & \text{constant} \end{array} \\
\left[\begin{array}{ccccc|c}
2 & -3 & 0 & 1 & 0 & 18 \\
6 & 5 & 1 & 0 & 0 & 50 \\
\hline
-4 & -1 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}$$

most negative

ratios
$18/2 = 9$
$50/6 = 8\frac{2}{6}$ ← smallest non-negative ratio

Pivot is 6 in R2, C1

(b)
$$\begin{array}{c}
\begin{array}{cccccccc} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \end{array} \\
\left[\begin{array}{ccccccc|c}
1 & 0 & \frac{2}{5} & 0 & 0 & -\frac{5}{2} & 0 & 22 \\
0 & 6 & 3 & 0 & 1 & 2 & 0 & 0 \\
0 & \frac{3}{4} & 7 & 1 & 0 & 0 & 0 & 12 \\
\hline
0 & -2 & -9 & 0 & 0 & -4 & 1 & 340
\end{array}\right]
\end{array}$$

most negative

ratios
$22/\frac{2}{5} = 55$
$0/3 = 0$
$12/7$

Pivot is 3 in R2, C3

3. State the the value of each variable and whether the variable is basic or non-basic for the given simplex tableaus below. Then determine if the given simplex tableau is a final tableau.

(a)
$$\begin{array}{cccccc} x & y & s_1 & s_2 & P & \text{constant} \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & -3 & 1 & 1 & 0 & 18 \\ 1 & 5 & \frac{1}{3} & 0 & 0 & 50 \\ \hline 0 & 1 & \frac{1}{2} & 0 & 1 & 275 \end{array}\right]$$

$$\begin{array}{ccc} x & s_2 & P \end{array}$$
$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 18 \\ 1 & 0 & 0 & 50 \\ \hline 0 & 0 & 1 & 275 \end{array}\right]$$

Basic	Non-Basics
$x = 50$	$y = 0$
$s_2 = 18$	$s_1 = 0$
$P = 275$	

Final Tableau

(b)
$$\begin{array}{cccccccc} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \end{array}$$
$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{2}{5} & 0 & -\frac{5}{2} & 0 & 22 \\ 0 & 6 & 0 & 3 & 1 & 2 & 0 & 0 \\ 0 & \frac{3}{4} & 1 & 7 & 0 & 0 & 0 & 12 \\ \hline 0 & -2 & 0 & -9 & 0 & -4 & 1 & 340 \end{array}\right]$$

$$\begin{array}{cccc} x & z & s_2 & P \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 22 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 12 \\ \hline 0 & 0 & 0 & 1 & 340 \end{array}\right]$$

Basic	Non-Basic
$x = 22$	$y = 0$
$z = 12$	$s_1 = 0$
$s_2 = 0$	$s_3 = 0$
$P = 340$	

Not the Final Tableau

4. Solve the linear programming problem, using the Simplex Method, if possible.

Maximize: $P = 9x + 7y + 11z$

Subject to: $-2x + 4y - 6z \leq 1200$

$8x + y + 3z \leq 800$

$5x - 10y \geq -10$

$x \geq 0,\ y \geq 0,\ z \geq 0$

$-2x + 4y - 6z + s_1 = 1200$

$8x + y + 3z + s_2 = 800$

$-5x + 10y + s_3 = 10$

$-9x - 7y - 11z + P = 0$

$$\begin{array}{ccccccc} x & y & z & s_1 & s_2 & s_2 & P \end{array}$$
$$\left[\begin{array}{ccccccc|c} -2 & 4 & -6 & 1 & 0 & 0 & 0 & 1200 \\ 8 & 1 & 3 & 0 & 1 & 0 & 0 & 800 \\ -5 & 10 & 0 & 0 & 0 & 1 & 0 & 10 \\ \hline -9 & -7 & -11 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

—

$800/3 \leftarrow$

—

$$\left[\begin{array}{ccccccc|c} 14 & 6 & 0 & 1 & 2 & 0 & 0 & 2800 \\ 8/3 & 1/3 & 1 & 0 & 1/3 & 0 & 0 & 800/3 \\ -5 & 10 & 0 & 0 & 0 & 1 & 0 & 10 \\ \hline 61/3 & -\frac{10}{3} & 0 & 0 & 11/3 & 0 & 1 & 8800/3 \end{array}\right]$$

$2800/6$

$800/3 / 1/3 = 800$

$10/10 = 1 \leftarrow$

$$\begin{array}{ccccccc} x & y & z & s_1 & s_2 & s_3 & P \end{array}$$
$$\left[\begin{array}{ccccccc|c} 17 & 0 & 0 & 1 & 2 & -3/5 & 0 & 2794 \\ 17/6 & 0 & 1 & 0 & 1/3 & -1/30 & 0 & 799/3 \\ -1/2 & 1 & 0 & 0 & 0 & 1/10 & 0 & 1 \\ \hline 56/3 & 0 & 0 & 0 & 11/3 & 1/3 & 1 & \frac{8810}{3} \end{array}\right]$$

$x = 0$
$y = 1$
$z = \frac{799}{3}$
$P = \frac{8810}{3}$

Solution: $(x, y, z) = (0, 1, \frac{799}{3})$

$P = \frac{8810}{3}$

5. Use the Simplex Method, if possible, to solve the lienar programming problem.

You have \$12,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than \$9,000. If Stock A earn 3% annual interest, and Stock B earn 4% annual interest, how much money should you invest in each to maximize your annual interest?

$a :=$ the amount, in dollars, invested in Stock A

$b :=$ the amount, in dollars, invested in Stock B

$I :=$ the interest earned, in dollars

Maximize: $I = 0.03a + 0.04b$

Subject to:

$a + b \le 12000$ → $a + b + s_1 = 12000$

$a \ge 2b$ → $-a + 2b + s_2 = 0$

$a \le 9000$ → $a + s_3 = 9000$

$-0.03a - 0.04b + I = 0$

$$\begin{array}{cccccc|c|c} a & b & s_1 & s_2 & s_3 & I & & \\ 1 & 1 & 1 & 0 & 0 & 0 & 12000 & 12000 \\ -1 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \leftarrow \\ 1 & 0 & 0 & 0 & 1 & 0 & 9000 & — \\ \hline -0.03 & -0.04 & 0 & 0 & 0 & 1 & 0 & \\ & \uparrow & & & & & & \end{array}$$

$$\begin{array}{cccccc|c|c} \frac{3}{2} & 0 & 1 & -\frac{1}{2} & 0 & 0 & 12000 & 8000 \leftarrow \\ -\frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & — \\ 1 & 0 & 0 & 0 & 1 & 0 & 9000 & 9000 \\ \hline -\frac{1}{20} & 0 & 0 & \frac{1}{50} & 0 & 1 & 0 & \\ \uparrow & & & & & & & \end{array}$$

$$\begin{array}{cccccc|c} a & b & s_1 & s_2 & s_3 & I & \\ 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 & 8000 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 4000 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{3} & 1 & 0 & 1000 \\ \hline 0 & 0 & \frac{1}{30} & \frac{1}{300} & 0 & 1 & 400 \end{array}$$

$a = 8000$

$b = 400$

$I = 400$

The maximum interest you can earn is \$400 when you invest \$8000 in Stock A and \$400 in Stock B.

1. State the sample space for each experiment:

(a) Selecting a letter at random from the word "skate" and noting the letter.

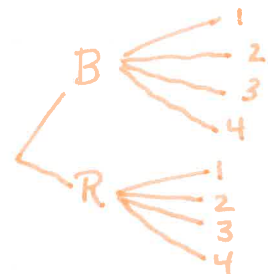
$S = \{s, k, a, t, e\}$

(b) A standard 30-sided die is rolled and it is noted whether the number is a multiple of 4 or is not a multiple of 4.

$S = \{\text{multiple of 4}, \text{not a multiple of 4}\}$

(c) A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.

$S = \{(B,1), (B,2), (B,3), (B,4)$
$(R,1), (R,2), (R,3), (R,4)\}$



2. Consider the experiment of selecting a letter at random from the word "skate" and noting the letter.

(a) State the certain event for the experiment.

$C = \{s, k, a, t, e\}$

(b) Given an example of an impossible event for the experiment.

$I = \{v\}$ The event "a 'v' is drawn"

(c) Write the outcomes in the event, $J :=$ the event "a consonant is drawn."

$J = \{s, k, t\}$

3. A card is drawn from a standard deck of 52-cards, noting the color, and then a standard four-sided die is rolled, noting the number facing uppermost.

(a) State all the simple events for the experiment.

$\{(B,1)\}$ $\{(R,1)\}$
$\{(B,2)\}$ $\{(R,2)\}$
$\{(B,3)\}$ $\{(R,3)\}$
$\{(B,4)\}$ $\{(R,4)\}$

(b) State the total number of possible events.

$n = 8$

2^8 possible events

(c) Write the outcomes in the event, $M :=$ the event "a black or a number less an 3 is rolled."

$M = \{(B,1), (B,2), (B,3), (B,4), (R,1), (R,2)\}$

4. An experiment consists of rolling a five-sided, noting the number showing uppermost and then spinning a spinner with five equal regions (red, blue, purple, maroon, and green), noting the color.

Let

- $V :=$ the event "a number greater than 3 is rolled"
- $W :=$ the event "an even is rolled"
- $X :=$ the event "the spinner lands on blue"
- $Y :=$ the event "the spinner lands on a color other than green"
- $Z :=$ the event "the spinner lands on purple or maroon."

(a) Write the symbolic notation for the event, H, that "a number less than or equal to 3 is rolled or the spinner lands on a color other than green, but not blue."

$$H = V^c \cup Y \cap X^c$$

(b) Describe the event $Z \cup Y \cup W^C$

The event "the spinner lands on purple or maroon or a color other than green or an odd is rolled on the die".

(c) Are event V and event W mutually exclusive? Explain why or why not.

$V = \{(4,r), (4,b), (4,p), (4,m), (4,g), (5,r)\ (5,b), (5,p), (5,m), (5,g)\}$

$W = \{(2,r), (2,b), (2,p), (2,m), (2,g), (4,r), (4,b), (4,p), (4,m), (4,g)\}$

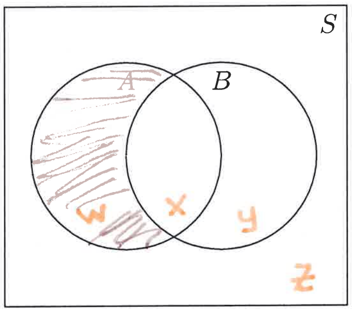
No, because they have overlap.

$V \cap W = \{(4,r), (4,b), (4,p), (4,m), (4,g)\}$

5. Let A and B be two events of the sample space, S.

Use a two-circle Venn diagram to illustrate which region(s) contain the outcomes of the resulting events.

a. $B^C \cap A$



$A = \{w, x\}$

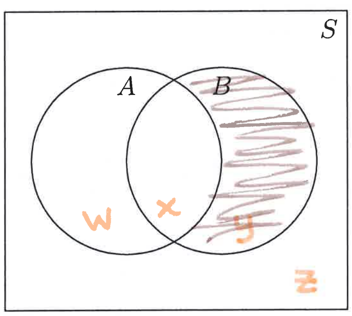
$B = \{x, y\} \Rightarrow B^c = \{w, z\}$

$B^c \cap A = \{w, z\} \cap \{w, x\}$

in both

$B^c \cap A = \{w\}$

b. $(A \cup B) \cap A^C$



$A = \{w, x\} \Rightarrow A^c = \{y, z\}$

$B = \{x, y\}$

$(A \cup B) = \{w, x\} \cup \{x, y\}$

combine

$A \cup B = \{w, x, y\}$

$(A \cup B) \cap A^c = \{w, x, y\} \cap \{y, z\}$

in both

$(A \cup B) \cap A^c = \{y\}$