SECTION 4.2: BASICS OF PROBABILITY

Pr 1. Determine if the sample space for each experiment is uniform or not.

(a) Selecting a letter at random from the word "skate" and noting the letter.

Each letter occurs once, so the probability of each is the same.
Yes it is uniform

(b) A standard 30-sided die is rolled and it is noted whether the number is a multiple of 4 or is not a multiple of 4.

There are only 7 of the 30 outcomes that are a multiple of 4, so no it is not uniform

Pr 2. A card is selected from a well-shuffled standard 52-card deck. Compute each of the following probabilities.

(a) P(a 5 is drawn) $= \frac{4}{52}$

(b) P(a red card is drawn) $= \frac{26}{52}$

(c) P(a black Ace is drawn) $= \frac{2}{52}$

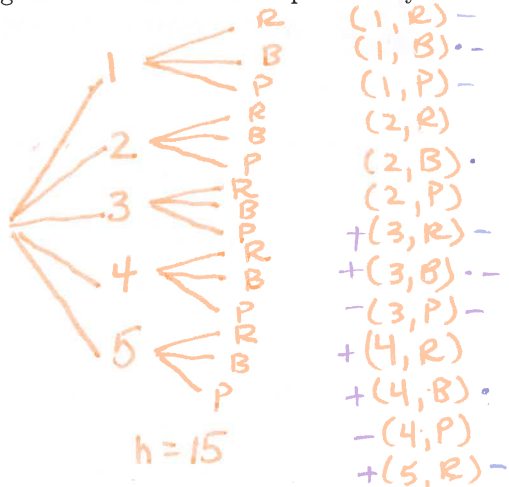
(d) P(a card which is not a diamond is drawn) $= \frac{39}{52}$

(e) P(a face card is drawn or spade is drawn) $= \frac{19}{52}$

Pr 3. An experiment consists of rolling a five-sided, noting the number showing uppermost and then spinning a spinner with three equal regions (red, blue, and purple), noting the color. What is the probability that

(a) A 3 is rolled

$P(3 \text{ is rolled}) = \frac{3}{15}$



(b) The spinner lands on a color other than maroon.

$P(\text{spinner lands on a color other maroon}) = \frac{15}{15}$

(c) An odd number is rolled or the spinner lands on ~~green~~ blue.

$P(\text{odd number is rolled or spinner lands on blue}) = \frac{11}{15}$

(d) A number greater than 2 and the spinner does not land purple.

$P(\text{number greater than 2 and spinner does not land on purple}) = \frac{6}{15}$

Pr 4. A music store selected 1000 customers at random and surveyed them to determine a relationship between age of purchaser and monthly purchases of cds. The results are given in the table below.

	0	1	2	3	4 or More	**Totals**
Under 12 (A)	50	60	30	20	10	170
12 - 18 (B)	30	100	90	30	40	290
19 - 25 (C)	70	110	100	30	20	330
Over 25 (D)	100	50	40	10	10	210
Totals	250	320	260	90	80	1000

If a surveyed person is selected at random, compute each of the following.

(a) $P(C \cap 3)$ = $\frac{30}{1000}$ (and)

(b) $P(A \cup 1)$ = $\frac{430}{1000}$ (combine)

$50+60+30+20+10+100+110+50$

(c) $P((B \cup D)^C \cap 4)$ = $\frac{30}{1000}$

$(B \cup D)^C = A \cup C$

Pr 5. Is the following probability distribution valid? If valid, does the distribution represent an experiment with uniform sample space?

Outcomes	-3	0	3	6	9
Probability	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{10}{25}$	$\frac{8}{25}$	$\frac{4}{25}$

$\frac{2}{25}+\frac{1}{25}+\frac{10}{25}+\frac{8}{25}+\frac{4}{25}$

Yes it is valid,

but not uniform

Section 4.3: Rules of Probability

1. Let $\mathbf{S} = \{s_1, s_2, s_3, s_4\}$ be the sample space for an experiment with the distribution given below.

Outcome	s_1	s_2	s_3	s_4
Probability	$\frac{3}{50}$	$\frac{4}{25}$		$\frac{18}{50}$

$\frac{3}{50} + \frac{8}{50} + \frac{18}{50} = \frac{29}{50}$

Let $A = \{s_1, s_3\}$ and $B = \{s_1, s_4\}$.

(a) Fill in the missing probability in the distribution table.

$P(s_3) = 1 - \frac{29}{50} = \frac{21}{50}$

Determine the following probabilities.

(b) $P(A) = \frac{3}{50} + \frac{21}{50} = \frac{24}{50}$

(c) $P(A \cup B) = 1 - P(s_2) = 1 - \frac{4}{25} = \frac{21}{25}$

s_1, s_3, s_4

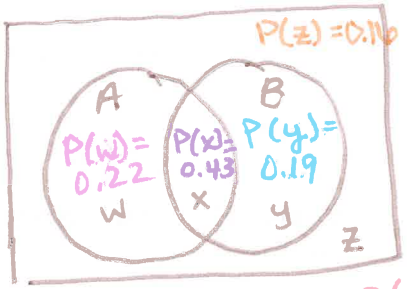
(d) $P\left((A \cap B)^C\right) = 1 - P(s_1) = 1 - \frac{3}{50} = \frac{47}{50}$

$A \cap B = \{s_1\}$

2. Let A and B be two events of an experiment. Suppose $P(A) = 0.65, P(B) = 0.62$, and $P(A \cup B) = 0.84$. Calculate the following probabilities:

(a) $P\left(A^C\right) = 1 - P(A)$
$= 1 - 0.65$
$= 0.35$

(b) $P(A \cap B) = 0.43$



$A = \{w, x\}$
$B = \{x, y\}$
$A \cup B = \{w, x, y\}$

$P(A) = P(w) + P(x) = 0.65$
$P(w) = 0.65 - 0.43$
$P(w) = 0.22$

$P(B) = P(x) + P(y) = 0.62$
$P(y) = 0.62 - 0.43$
$P(y) = 0.19$

(c) $P\left(A^C \cup B^C\right) = 1 - P(x)$
$= 1 - 0.43$
$= 0.57$

$A^C = \{y, z\}$
$B^C = \{w, z\}$
$A^C \cup B^C = \{w, y, z\}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.84 = 0.65 + 0.62 - P(A \cap B)$
$-0.43 = -P(A \cap B)$
$0.43 = P(A \cap B)$
$0.43 = P(x)$

Section 4.4: Probability Distributions

1. The probability distribution for tossing a coin three times and counting the number of tails is given below.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(a) Compute the probability that more than one head is tossed.

$P(\text{more than one head is tossed}) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

2, 3

(b) Compute the probability that four heads are tossed.

$P(\text{four heads are tossed}) = \frac{0}{8} = 0$

(c) State the expected number of heads in an experiment where three coins are tossed.

$E(x) = 0 \cdot \left(\frac{1}{8}\right) + 1 \cdot \left(\frac{3}{8}\right) + 2 \cdot \left(\frac{3}{8}\right) + 3 \cdot \left(\frac{1}{8}\right)$

$E(x) = \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$

$E(x) = \frac{12}{8}$

2. You are going on a European vacation and decide to purchase travel insurance on your brand new luggage worth \$1500. The insurance policy will cost \$48. In the event your luggage is damaged to the point of needing duct tape, then you will receive 50% of the value of the luggage. In the event your luggage is lost or stolen, then you will receive 100% of the value of the luggage. According to airline data, the probability of your luggage being damaged and needging duct tape is 1%, while the probability your luggage is lost or stolen is 0.8%. Let X be the insurance company's net gain or loss on the policy described. premium − payout

(a) Create a probability distribution for X.

	Damaged	Lost	Nothing
X	$48 - 0.5(1500) = -702$	$48 - 1500 = -1452$	48
$P(X)$	0.01	0.008	$1 - 0.01 - 0.008 = 0.982$

(b) Compute the insurance company's expected profit for this policy.

$E(x) = -702(0.01) + (-1452)(0.008) + 48(0.982)$

$= 28.5$

The expected profit is \$28.50.

$S = \{(K,r),(K,b),(K,g),(K,y),(D,r),(D,b),(D,g),(D,y)\}$ (K = black, D = red)

3. You play a game where a card is drawn from a well-shuffled standard deck of 52 cards, noting the color of the card, and a spinner divided into four equal regions (red, blue, green, and yellow) is spun, noting the color. If the spinner lands on a color other than yellow, you win \$3. If the color of the card is red and the spinner lands on red, you win \$10. Otherwise you lose. Let X be your winnings. What you win − what you paid (p)

a. Create a probability distribution for X.

X	Spinner not yellow	red and red	Lose
	$3 - p$	$10 - p$	$0 - p$
$P(X)$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{2}{8}$

Do not include (D,r) here

b. Compute your expected winnings for the game.

$$E(x) = \frac{5}{8}(3-p) + \frac{1}{8}(10-p) + \frac{2}{8}(0-p)$$

$$= \frac{15}{8} - \frac{5}{8}p + \frac{10}{8} - \frac{1}{8}p - \frac{2}{8}p$$

$$E(x) = \frac{25}{8} - p$$

c. How much should be charged in order to make the game fair?

$E(x) = 0$

$$E(x) = \frac{25}{8} - p$$

$$0 = \frac{25}{8} - p$$

$$p = \frac{25}{8} = 3.125$$

You should pay \$3.125 for the game to be fair.

This is not possible so the game can not be fair.