SECTION 5.1: RELATIONS AND FUNCTIONS

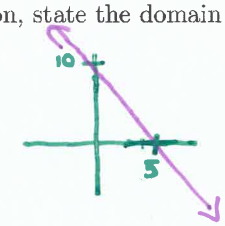
Pr 1. State the inputs and outputs of $R_1 = \{(10, -23), (-8, 41), (-12, 41), (36, 36)\}$

Input : $\{10, -8, -12, 36\}$

Output : $\{-23, 41, 36\}$

Pr 2. Determine if the given relation is a function. If the relation is a function, state the domain and range of the function.

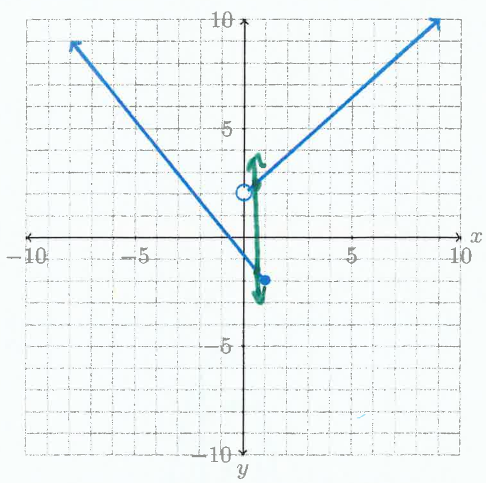
(a) $2x + y = 10$ a line => a function



Domain: $x \in (-\infty, \infty)$

Range: $y \in (-\infty, \infty)$

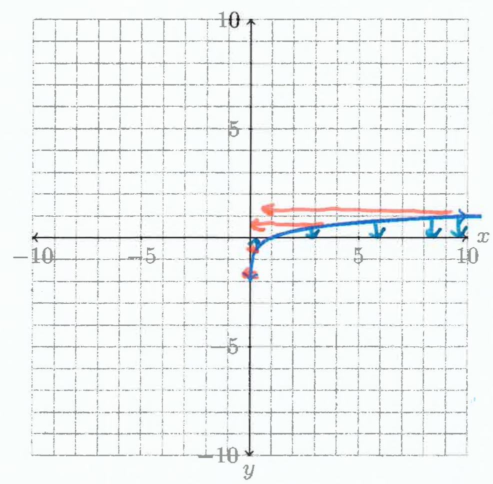
(b)



Not a function

Does not pass the Vertical Line Test

(c)

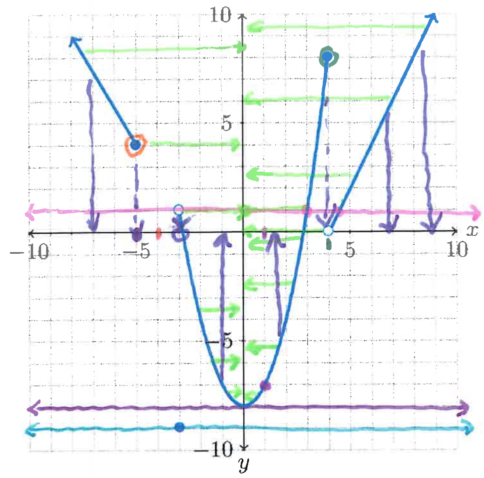


A function

Domain: $x \in (0, \infty)$

Range: $y \in (-\infty, \infty)$

Pr 3. Use the graph of $g(x)$ below to answer each of the following.



(a) $g(-4)$ = does not exist

(b) $g(1)$ = -7

(c) $g(4)$ = 8

(d) $g(-5)$ = 4

(e) $g(x) = 1$ $x = 3$ and 4.5

(f) $g(x) = -8$ $x = 0$

(g) $g(x) = -9$ $x = -3$

(h) State the domain of $g(x)$.

$x \in (-\infty, -5] \cup (-3, \infty)$

(i) State the range of $g(x)$.

$y \in [-8, \infty)$

Pr 4. Use the function $f(x) = 4 - 5x$ to compute and expand and simplify each of the following.

(a) $f(3q)$

$$f(3q) = 4 - 5(3q)$$
$$= 4 - 15q$$

(b) $f(x - 10)$

$$f(x-10) = 4 - 5(x - 10)$$
$$= 4 - 5x + 50$$
$$= 54 - 5x$$

(c) $f(x) - f(10)$

$$f(x) - f(10) = (4 - 5x) - (4 - 5(10))$$
$$= 4 - 5x - 4 + 50$$
$$= 50 - 5x$$

Pr 5. Use the function $g(x) = 8x^2 - 9x$ to compute and expand and simplify each of the following.

(a) $g(-4)$

$$g(-4) = 8(-4)^2 - 9(-4)$$
$$= 8(16) + 36$$
$$= 128 + 36$$
$$= 164$$

(b) $g(x+h)$

$$g(x+h) = 8(x+h)^2 - 9(x+h)$$
$$= 8(x^2 + 2xh + h^2) - 9x - 9h$$
$$= 8x^2 + 16xh + 8h^2 - 9x - 9h$$

(c) $g(x+h) - g(x)$

$$g(x+h) - g(x) = \left[8(x+h)^2 - 9(x+h)\right] - \left[8x^2 - 9x\right]$$
$$= \left[8x^2 + 16xh + 8h^2 - 9x - 9h\right] - 8x^2 + 9x$$
$$= 16xh + 8h^2 - 9h$$

Pr 1. Determine if the given function is a polynomial function. If the answer is yes, state the degree, leading coefficient, and constant term.

(a) $f(x) = -40x + 4x^3 - 6x^{3.8}$

not a polynomial

(b) $g(w) = 3w^2 - w^3 + 7w - \sqrt{21}$ $= \underbrace{-w^3}_{\text{leading term}} + 3w^2 + 7w \underbrace{-\sqrt{21}}_{\text{constant term}}$

polynomial

degree: $3 = n$

leading coefficient: $a_n = -1$

constant term: $a_0 = -\sqrt{21}$

Pr 2. Describe the end behavior of each polynomial function, both symbolically and with a quick sketch of the end behavior.

(a) $f(x) = \boxed{-3x^4} + 3x^3 - 16x - 8^5$

$n = 4$ - even

$a_n = -3 < 0$

As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to -\infty$

(b) $g(x) = 12x^4 - 9 \boxed{+ 18x^7} - x^2$

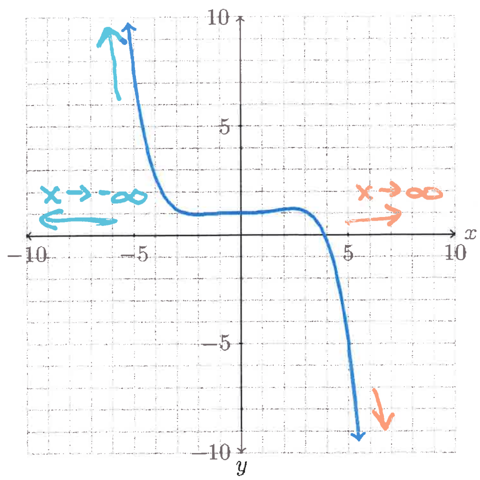
$n = 7$ - odd

$a_n = 18 > 0$

As $x \to -\infty$, $g(x) \to -\infty$

As $x \to \infty$, $g(x) \to \infty$

Pr 3. Describe the end behavior symbolically for the polynomial function, $f(x)$, graphed below.



As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to -\infty$

Pr 4. State the domain of each polynomial function.

(a) $f(x) = 4x^3 - 6x^2 - 40x$

a polynomial $\Rightarrow$ Domain: $x \in (-\infty, \infty)$

(b) $g(w) = 3w^2 - w^3 + 7w - 21$

a polynomial $\Rightarrow$ Domain : $w \in (-\infty, \infty)$

Pr 5. Determine all exact real zeros, the x-intercept(s), and y-intercept of each given polynomial function, if possible.

(a) $g(x) = 4x^3 - 6x^2 - 40x = 2x(2x+5)(x-4)$

Real zeros: $2x(2x+5)(x-4) = 0$

$2x = 0$ or $2x+5 = 0$ or $x-4=0$

$x = 0$ or $x = -\frac{5}{2}$ or $x = 4$

Exact zeros: $x = 0, -\frac{5}{2}, 4$

x-intercepts: $(0,0), (-\frac{5}{2}, 0), (4, 0)$

y-intercepts: $(0, 0)$

y-int: $2(0)[2(0)+5][(0)-4] = g(0)$

$g(0) = 0(5)(-4)$

$= 0$

(b) $h(w) = 3w^2 - w^3 + 7w - 21$

Real zeros: $-w^3 + 3w^2 + 7w - 21 = 0$

$-w^2(w-3) + 7(w-3) = 0$

$(-w^2+7)(w-3) = 0$

$-w^2+7 = 0$ or $w-3=0$

$w^2 = +7$ $\quad$ $w = 3$

$w = \pm\sqrt{7}$

Exact zeros: $w = \pm\sqrt{7}, 3$

x-intercepts: $(-\sqrt{7}, 0), (\sqrt{7}, 0), (3, 0)$

y-intercepts: $(0, -21)$

y-int: $h(0) = 3(0)^2 - (0)^3 + 7(0) - 21$

$h(0) = 0 - 0 + 0 - 21$

$= -21$

(c) $k(x) = (x^2+4)(x^2-9)$

Real zeros: $(x^2+4)(x^2-9) = 0$

$x^2+4 = 0$ or $x^2-9 = 0$

no real zeros $\quad$ $(x-3)(x+3) = 0$

$x-3 = 0$ or $x+3 = 0$

$x = 3$ or $x = -3$

Exact zeros: $x = -3, 3$

x-intercepts: $(-3, 0), (3, 0)$

y-intercepts: $(0, -36)$

y-int: $k(0) = [(0)^2+4][(0)^2 - 9]$

$k(0) = (4)(-9)$

$= -36$

Pr 6. $h(x) = 49 - 81x^2 = -81x^2 + 49$

$a = -81,\ b = 0,\ c = 49$

vertex: $(0, 49)$

axis of symmetry: $x = 0$

domain: $x \in (-\infty, \infty)$

range: $y \in (-\infty, 49]$

x-intercept(s): $\left(\frac{7}{9}, 0\right)$ and $\left(-\frac{7}{9}, 0\right)$

y-intercept: $(0, 49)$

maximum value: 49

minimum value: None

Vertex: $x = \frac{-b}{2a}$

$x = \frac{-0}{2(-81)} = 0$

$h(0) = -81(0)^2 + 49 = 49$

x-int: $-81x^2 + 49 = 0$

$(-9x+7)(9x+7) = 0$

$x = 7/9$ or $x = -7/9$

y-int: $h(0) = -81(0)^2 + 49 = 49$

Pr 7. $j(x) = \frac{1}{5}x^2 + \frac{49}{500}x - \frac{31}{100}$

$a = \frac{1}{5},\ b = \frac{49}{500},\ c = -\frac{31}{100}$

vertex: $\left(\frac{49}{200}, -\frac{64401}{200000}\right)$

axis of symmetry: $x = 49/200$

domain: $x \in (-\infty, \infty)$

range: $y \in \left[-\frac{64401}{200000}, \infty\right)$

x-intercept(s): $\left(\frac{-49+\sqrt{64401}}{200}, 0\right)$ and $\left(\frac{-49-\sqrt{64401}}{200}, 0\right)$

y-intercept: $(0, -31/100)$

maximum value: none

minimum value: $-\frac{64401}{200000}$

Vertex: $x = \frac{-b}{2a}$

$x = \frac{-49/500}{2(1/5)} = -\frac{49}{200}$

$j\left(\frac{49}{200}\right) = \frac{1}{5}\left(\frac{49}{200}\right)^2 + \left(\frac{49}{500}\right)\left(\frac{49}{200}\right) - \frac{31}{100}$

$= \frac{-2401}{200000} - \frac{62000}{200000} = \frac{-64401}{200000}$

x-int: $\frac{1}{5}x^2 + \frac{49}{500}x - \frac{31}{100} = 0$

$x = \frac{-\left(\frac{49}{500}\right) \pm \sqrt{\left(\frac{49}{500}\right)^2 - 4\left(\frac{1}{5}\right)\left(\frac{-31}{100}\right)}}{2\left(\frac{1}{5}\right)}$

$x = \frac{-49 \pm \sqrt{64401}}{200}$

y-int: $j(0) = \frac{1}{5}(0)^2 + \frac{49}{500}(0) - \frac{31}{100}$

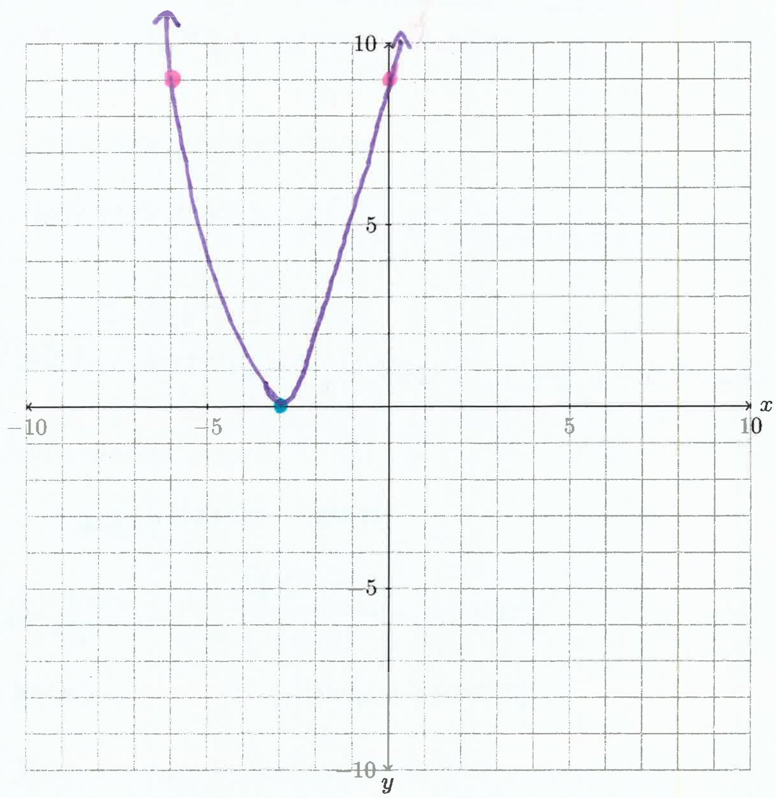
Pr 8. Graph the quadratic function with the following properties

i. As $x \to -\infty$, $h(x) \to \infty$ and as $x \to \infty$, $h(x) \to \infty$ ↖ ↗

ii. $h(x)$ has a single real zero of -3. $\Rightarrow$ $(-3, 0)$ is on graph

iii. There is a minimum value of 0. $\Rightarrow$ $(-3, 0)$ is the vertex

iv. The graph has a y-intercept of $(0, 9)$. $\Rightarrow$ $(-6, 9)$ on graph



Pr 9. Given $R(x) = -10x^2 + 3700x$ and $C(x) = 200x + 66000$, where x is the number of items made and sold, to determine each of the following. Assume both revenue and cost are given in dollars.

i. The number of items sold when revenue is maximized.

x vertex

$R(x) = -10x^2 + 3700x$

$$x = \frac{-b}{2a} = \frac{-3700}{2(-10)} = 185$$

185 items sold

ii. The maximum revenue.

y. of vertex

$R(185) = -10(185)^2 + 3700(185) = 342250$

Maximum revenue is \$342,250

iii. The number of items sold when profit is maximized.

x vertex

$P(x) = R(x) - C(x) = (-10x^2 + 3700x) - (200x + 66000) = -10x^2 + 3500x - 66000$

$$x = \frac{-b}{2a} = \frac{-(3500)}{2(-10)} = 175$$

175 items sold

iv. The maximium profit.

y of vertex

$P(175) = -10(175)^2 + 3500(175) - 66000$
$= 240250$

Maximum profit is \$240,250

v. The break-even quantity/quantities.

$P(x) = 0$ or $R(x) = C(x)$

$P(x) = -10x^2 + 3500x - 66000 = 0$

$$x = \frac{-(3500) \pm \sqrt{(3500)^2 - 4(-10)(-66000)}}{2(-10)} = \frac{-3500 \pm 3100}{-20} = 20 \text{ or } 330$$

20 items or 330 items

↓ start to make profit at 21 item

↓ lose a profit at 331 items

Pr 10. The cost to produce bottled spring water is given by $C(x) = 16x + 7400$, where x is the number of thousands of bottles produced. The profit from the sale of these bottles is given by the function

$P(x) = -x^2 + 310x - 7400.$

(a) How many bottles must be sold to maximize the $\boxed{\text{profit?}}$

x Vertex

$$P(x) = -x^2 + 310x - 7400$$

$$x = -\frac{b}{2a} = -\frac{310}{2(-1)} = 155$$

155 bottles

(b) What is the maximum $\boxed{\text{profit?}}$

y of vertex

$$P(155) = -(155)^2 + 310(155) - 7400$$
$$= 16625$$

Maximum profit is \$16,625

(c) What is the revenue when the $\boxed{\text{profit}}$ is maximized?

R(x) where x is from vertex of profit

$x = 155$

$$R(x) = P(x) + C(x)$$

$$R(155) = [-(155)^2 + 310(155) - 7400] + [16(155) + 7400]$$

$$R(155) = 26505$$

Need a revenue of \$26,505 to maximize the profits.

Pr 11. The cost of manufacturing collectible bobble head figurines is given by $C(x) = 60x + 375$, where x is the number of collectible bobble head figurines produced. If each figurine has a price-demand function of $p(x) = -1.6x + 300$, in dollars, determine

(a) the company's profit function.

$P(x) = R(x) - C(x)$ and $R(x) = \text{price} \cdot \text{quantity}$

$R(x) = (-1.6x + 300)x = -1.6x^2 + 300x$

$P(x) = [-1.6x^2 + 300x] - [60x + 375]$

$P(x) = -1.6x^2 + 240x - 375$

(b) how many figurines must be sold in order to maximize $\boxed{\text{revenue?}}$

x vertex

$R(x) = -1.6x^2 + 300x$

$x = \frac{-b}{2a} = \frac{-300}{2(-1.6)} = 93.75$

93.75 figurines

(c) how many figurines must be sold in order to maximize $\boxed{\text{profit?}}$

x vertex

$P(x) = -1.6x^2 + 240x - 375$

$x = \frac{-b}{2a} = \frac{-(240)}{2(-1.6)} = 75$

75 figurines

(d) at what price per figurine will the maximum $\boxed{\text{profit}}$ be achieved?

p(x) for x from vertex

$p(x) = -1.6x + 300$

and $x = 75$

$p(75) = -1.6(75) + 300$

$= 180$

\$180 is the price per figurine