Math 151- Week-In-Review 10

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Anti-derivatives

1. Find $f(x)$ given the following conditions.
   (a) $f'(x) = x^4(3x^2 - 5)$

   (b) $f'(x) = \dfrac{x^3 - 4}{x^4}$

   (c) $f'(x) = \cos(x)(\tan(x) + \sec(x))$

(d) $f'(x) = \dfrac{5}{x} - 7^x + \sqrt[5]{x^3} + \dfrac{2}{\sqrt{1-x^2}}$

(e) $f'(x) = 4(1+x^2)^{-1} - \csc^2(x) + \dfrac{1}{8x^3}$

(f) $f'(x) = 3\cos(x) - 5x^3 + e^x$ with $f(0) = 7$

(g) $f''(x) = 5x^4 - 6$ with $f'(0) = 4$ and $f(1) = 2$

(h) $f''(x) = \dfrac{4x^4 - 6}{x^2}$ with $f(1) = \dfrac{4}{3}$ and $f(3) = 30$

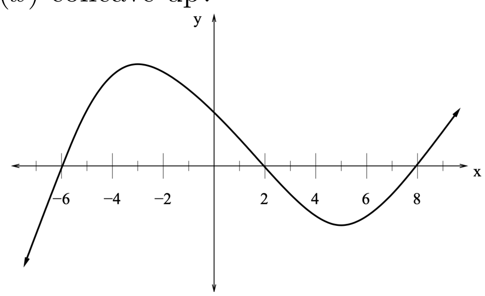
2. Approximate the area under the graph of $f(x) = 7x - x^2$ from $x = -1$ to $x = 5$ using 6 equal-width subintervals and using right endpoints.

3. Find an expression for the approximate area under the graph of $f(x) = \sqrt[3]{x^3 - x^2 + 9}$ on the interval $[-2, 7]$ using left endpoints.

4. Find an expression for the actual area under the graph of $f(x) = \dfrac{x^2}{3x - 2}$ on the interval $[-5, 2]$ using left endpoints. Do not evaluate the limit.

Tiny Test Review

1. The graph shown below is the graph of the second derivative of $f(x)$, that is $f''(x)$. Where is $f(x)$ concave up?



(a) $(1, \infty)$

(b) $(-\infty, -3) \cup (3, \infty)$

(c) $(-\infty, 1)$

(d) $(-\infty, -6) \cup (2, 8)$

(e) $(-6, 2) \cup (8, \infty)$

2. Find the value of the limit: $\lim_{x \to 0} \dfrac{2^{3x} - 5^x}{4x}$.

(a) 0

(b) $\dfrac{1}{4} \ln\left(\dfrac{8}{5}\right)$

(c) $\dfrac{1}{4} \ln\left(\dfrac{2}{5}\right)$

(d) 1

(e) $\dfrac{1}{4} \ln(10)$

3. Find all critical numbers for $f(x) = \sqrt[3]{x^2 - x - 2}$.

(a) $x = 2$ and $x = -1$

(b) $x = \dfrac{1}{2}$

(c) $x = 2$, $x = -1$ and $x = \dfrac{1}{2}$

(d) $x = -2$, $x = 1$ and $x = 2$

(e) $x = -2$, $x = 1$ and $x = \dfrac{1}{2}$

4. Find the intervals where $f(x) = \dfrac{\ln x}{x}$ is increasing or decreasing.

(a) $f(x)$ is increasing on the interval $\left(0, \dfrac{1}{e}\right)$ and decreasing on the interval $\left(\dfrac{1}{e}, \infty\right)$

(b) $f(x)$ is increasing on the interval (e, ∞) and decreasing on the interval $(0, e)$

(c) $f(x)$ is increasing on the interval $\left(\dfrac{1}{e}, \infty\right)$ and decreasing on the interval $\left(0, \dfrac{1}{e}\right)$

(d) $f(x)$ is increasing on the interval $(0, e)$ and decreasing on the interval (e, ∞)

(e) $f(x)$ is always increasing.

5. Find the absolute maximum and absolute minimum for $f(x) = x^2 + \dfrac{2}{x}$ over the interval $\left[\dfrac{1}{2}, 2\right]$.

(a) Absolute maximum: $y = 5$, Absolute minimum: $y = 3$

(b) Absolute maximum: $y = 4$, Absolute minimum: $y = 3$

(c) Absolute maximum: $y = 5$, Absolute minimum: $y = -1$

(d) Absolute maximum: $y = 4.5$, Absolute minimum: $y = -1$

(e) Absolute maximum: $y = 4.5$, Absolute minimum: $y = 3$

6. Find a number c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 + x - 1$ on the interval $[0, 2]$.

(a) $\sqrt{\dfrac{5}{3}}$

(b) 1

(c) $\sqrt{3}$

(d) $\dfrac{2}{\sqrt{3}}$

(e) None of these.

7. Given $f(x) = 2xe^{3x}$, find the intervals where is f(x) increasing or decreasing.

Find the local maximum and/or minimum for f(x), if any.

Find the intervals of concavity for f(x).

Find the point(s) of inflection for f(x), if any.

8. A rectangle is bounded by the x-axis and the semicircle $f(x) = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

9. Given that $\mathbf{r}''(t) = \langle e^t + t, \cos(t) - 1 \rangle$, $\mathbf{r}'(0) = \langle 1, -2 \rangle$ and $\mathbf{r}(0) = \langle 4, 12 \rangle$, find $\mathbf{r}(t)$.

10. Which of the following is an antiderivative of $f(x) = xe^x$?
    (a) $(x + 1)e^x + C$

    (b) $(x - 1)e^x + C$

    (c) $\frac{1}{2}x^2e^x + C$

    (d) $-\frac{1}{2}x^2e^x + C$

    (e) None of these.

