Math 151- Week-In-Review 12

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Final Exam Review

1. For a continuous function f, if $f'(3) = 0$ and $f''(3) = 7$, which of these statements do we know to be true about the graph of f at $x = 3$?

   (a) There is a local maximum at $x = 3$.

   (b) There is an absolute maximum at $x = 3$.

   (c) There is a local minimum at $x = 3$.

   (d) There is an absolute minimum at $x = 3$.

   (e) There is not enough information to determine the behavior of the graph at $x = 3$.

2. Find the x-values where local maximums or local minimums occur for $y = \dfrac{24}{x^2} + 12x + b$.

   (a) local min at $x = \sqrt[3]{4}$ only

   (b) local max at $x = 0$, local min at $x = \sqrt[3]{4}$

   (c) local min at $x = 0$, local max at $x = \sqrt[3]{4}$

   (d) local max at $x = \sqrt[3]{4}$ only

   (e) local max at $x = 0$ only

3. The function $f(x)$ is defined at all real numbers except 8 and $f'(x) = \dfrac{-7(x-1)(x+4)}{(x-8)^4}$. At what x-value does $f(x)$ have a local minimum?

   (a) $x = 8$ only

   (b) $x = 1$ only

   (c) $x = -4$ only

   (d) $x = 1$ and $x = 8$ only

   (e) $f(x)$ does not have a local maximum.

4. Evaluate $\lim_{x\to 0} \dfrac{e^{4x} - 5 - 4x}{x^2}$.

   (a) 0

   (b) 8

   (c) ∞

   (d) 2

   (e) $-\infty$

5. Approximate the area under the curve $f(x) = x^2 - 1$ on the interval $[2, 8]$ using three rectangles of equal width and midpoints.

   (a) 106

   (b) 226

   (c) 80

   (d) 304

   (e) 160

6. Rancher Xiaoyu wants to fence a new pasture along the using a straight river as one side of the boundary. If Rancher Xiaoyu has 1200 yards of fencing materials, what are the **DIMENSIONS** of the largest area of the pasture that Rancher Xiaoyu can enclose?

   (a) 300 yards x 300 yards

   (b) 300 yards x 600 yards

   (c) 250 yards x 700 yards

   (d) 90,000 square yards

   (e) 180,000 square yards

7. A particle has an acceleration given by $a(t) = 12t$ on the interval $[0, 10]$. If this particle has an initial velocity of 12 meters per second and has a position of 15 meters at $t = 1$, find the position at $t = 5$.

   (a) 339 meters

   (b) 315 meters

   (c) 325 meters

   (d) 311 meters

   (e) 301 meters

8. Let f be a differentiable function such that $f(3) = 1$ and $f'(3) = -3$. If $h(x) = \dfrac{2f(x)}{x^2+1}$, find $h'(3)$.

   (a) $-\dfrac{72}{100}$

   (b) $-\dfrac{48}{100}$

   (c) $\dfrac{72}{100}$

   (d) $-\dfrac{72}{10}$

   (e) $\dfrac{48}{10}$

9. Find the 4003^{rd} derivative of $g(x) = 2\sin(5x)$.

   (a) $2 \cdot 5^{4003} \cos(5x)$

   (b) $-2 \cdot 5^{4003} \sin(5x)$

   (c) $2 \cdot 5^{4003} \sin(5x)$

   (d) $2^{4003} \cdot 5^{4003} \sin(5x)$

   (e) $-2 \cdot 5^{4003} \cos(5x)$

10. Sand is being dropped at a rate of 10 ft^3/min onto a cone-shaped pile. If the height of the pile is always twice the base radius, at what rate is the height increasing when the pile is 8 ft high? Recall the volume formula for a cone is $V = \frac{\pi}{3}r^2h$.

(a) $\frac{5}{64\pi}$ ft/min

(b) $\frac{5}{8\pi}$ ft/min

(c) $\frac{5}{32\pi}$ ft/min

(d) $\frac{10}{9\pi}$ ft/min

(e) $\frac{10}{27\pi}$ ft/min

11. Find the value c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = -3x^2 + 5x + 5$ on the interval $[0, 3]$.

(a) $\frac{17}{6}$

(b) 3

(c) $\frac{3}{2}$

(d) 0

(e) $\frac{17}{18}$

12. Use logarithmic differentiation to find the derivative of $f(x) = \frac{(x^3+2x)^{400}}{(1+x)^{300}}$.

(a) $f'(x) = \left[\frac{400(3x^2+2)}{x^3+2x} - \frac{300}{1+x}\right] \cdot \frac{(x^3+2x)^{400}}{(1+x)^{300}}$

(b) $f'(x) = \left[\frac{400}{x^3+2x} - \frac{300}{1+x}\right] \cdot \frac{(x^3+2x)^{400}}{(1+x)^{300}}$

(c) $f'(x) = \frac{400(3x^2+2)}{x^3+2x} - \frac{300}{1+x}$

(d) $f'(x) = \frac{400(3x^2+2)(x^3+2x)^{398}}{(1+x)^{300}} - \frac{300(x^3+2x)^{400}}{(1+x)^{301}}$

(e) $f'(x) = \frac{100(x^3+2x)^{398}(9x^3+12x^2+2x+8)}{(1+x)^{301}}$

13. Find the t-values where the tangent line to the following parametrically defined curve is horizontal or vertical.

$$x = 2t^3 - t^2 + 6 \qquad \text{and} \qquad y = -t^3 + \frac{9}{2}t^2 - 6t$$

(a) horizontal tangents occur at $t = 1, 2$; vertical tangents occur at $t = 0, \frac{1}{3}$

(b) horizontal tangents occur at $t = 0, \frac{1}{3}$; vertical tangents occur at $t = 1, 2$

(c) horizontal tangents occur at $t = \frac{2}{3}, 1$; vertical tangents occur at $t = 0$

(d) horizontal tangents occur at $t = 0$; vertical tangents occur at $t = \frac{2}{3}, 1$

(e) horizontal tangents occur at $t = 1$; there are no vertical tangents

14. Which of the following is a vector of unit length tangent to $\left\langle \sqrt{10t+5},\ e^{4t-8} \right\rangle$ at $t = 2$?

(a) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

(b) $\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}} \right\rangle$

(d) $\langle 1, 1 \rangle$

(e) $\left\langle 1, \frac{4}{3} \right\rangle$

15. Find the values of the constants a and b that make the following piecewise function differentiable everywhere:

$$f(x) = \begin{cases} ax^3 + 16x & \text{if } x < 1 \\ 5x^2 + b & \text{if } x \geq 1 \end{cases}$$

(a) $a = 16,\ b = 5$

(b) $a = -2,\ b = 9$

(c) $a = -\frac{5}{3},\ b = \frac{28}{3}$

(d) $a = 1,\ b = 0$

(e) there is not enough information to determine a and b

16. Suppose $\int_5^9 g(x)\,dx = 4$. Evaluate $\int_5^9 (3 - 4g(x))\,dx$

(a) 39

(b) -13

(c) 19

(d) -4

(e) -36

17. Let $f(x) = \int_{\tan x}^{x} \frac{1}{\sqrt{4+t^3}}\,dt$. Find $f'(x)$

(a) $f'(x) = -\frac{\tan(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$

(b) $f'(x) = \frac{\sec^2(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$

(c) $f'(x) = -\frac{\sec^2(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$

(d) $f'(x) = \frac{\tan(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$

(e) $f'(x)$ does not exist

18. Evaluate $\int_1^2 \left(\frac{9}{x^5} - \frac{2}{x}\right) dx$

(a) $\frac{135}{64} - 2\ln(2)$

(b) $-\frac{135}{64} - \ln(2)$

(c) $\frac{135}{64} + 2\ln(2)$

(d) $-\frac{135}{64} - 2\ln(2)$

(e) $\frac{135}{64} + \ln(2)$

19. Evaluate $\int \left(3x^2 - 10 + \frac{3}{x^2+1}\right) dx$.

    (a) $\frac{x^3}{3} - 10x + 3\arctan(x) + C$

    (b) $\frac{x^3}{3} - 10x + 3\arcsin(x) + C$

    (c) $x^3 - 10x + 3\arctan(x) + C$

    (d) $x^3 - 10x + 3\arcsin(x) + C$

    (e) $\frac{x^3}{3} - 10x + 3\tan(x) + C$

20. The velocity function, in meters per second, is $v(t) = 3t - 7$. What is the displacement of the particle in the first four seconds it moves?

    (a) 4 m

    (b) −32 m

    (c) 32 m

    (d) 12 m

    (e) −4 m

21. A plane is flying at 850 mph at N45°E. The wind is blowing at 30mph S60°E. Find the true direction of the plane.

    (a) $\theta = \arctan\left(\frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} - 15}\right)$

    (b) $\theta = \arctan\left(\frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} + 15}\right)$

    (c) $\theta = \arctan\left(\frac{15 - 425\sqrt{2}}{15\sqrt{3} + 425\sqrt{2}}\right)$

    (d) $\theta = \arctan\left(\frac{425\sqrt{2} - 15}{425\sqrt{2} + 15\sqrt{3}}\right)$

    (e) $\theta = \arctan\left(\frac{15\sqrt{3} + 425\sqrt{2}}{15 - 425\sqrt{2}}\right)$

22. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of 60° above the horizontal. Find the work done on the box.

 (a) 50 ft-lb
 (b) $50\sqrt{3}$ ft-lb
 (c) $50\sqrt{2}$ ft-lb
 (d) 100 ft-lb
 (e) 10 ft-lb

23. Given the points $A(1,0)$, $B(0,2)$ and $C(3,4)$, find the angle, θ, located at the vertex A. That is, $\angle BAC$.

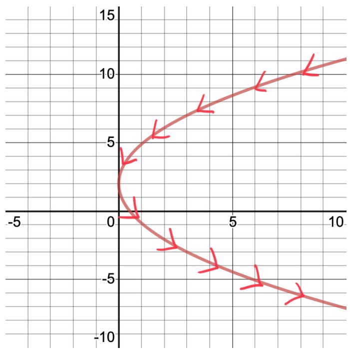
 (a) $\theta = \arccos\left(\frac{3}{5}\right)$
 (b) $\theta = \arccos\left(-\frac{1}{\sqrt{65}}\right)$
 (c) $\theta = 180°$
 (d) $\theta = \arccos\left(\frac{1}{\sqrt{65}}\right)$
 (e) $\theta = \arccos\left(\frac{3}{\sqrt{17}}\right)$

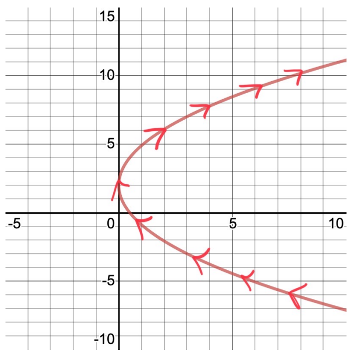
24. Find the slope of the tangent line to the graph $x^2y^2 - 3y = 0$ at the point $(1,-3)$.

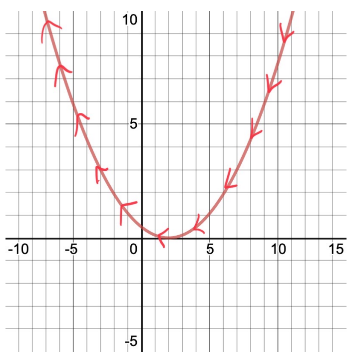
 (a) $-\frac{5}{2}$
 (b) $\frac{5}{2}$
 (c) -8
 (d) 2
 (e) -2

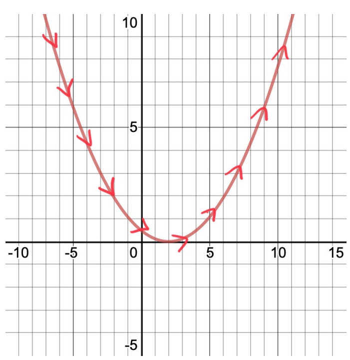
25. Which of the following graphs would match the parametric equations?

$$x = 3t^2 \text{ and } y = 2 - 5t$$

(a) 

(b) 

(c) 

(d) 

(e) None of the listed answers.

26. What is $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

(a) $\frac{5\pi}{6}$

(b) $-\frac{\pi}{6}$

(c) $-\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$

(e) $-\frac{5\pi}{6}$

27. Calculate $\lim_{x\to 3^+} \frac{x^2-2x-7}{x^2-5x+6}$.

(a) $-\infty$

(b) ∞

(c) 1

(d) -4

(e) $-\frac{7}{6}$

28. Use linear approximation to estimate $\sqrt[3]{27.2}$

(a) $\frac{801}{270}$

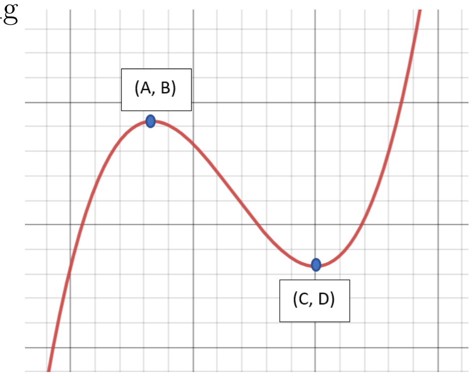
(b) $\frac{1}{135}$

(c) $\frac{406}{135}$

(d) $\frac{402}{135}$

(e) $\frac{803}{270}$

29. Given the graph of $f(x)$, on which of the following interval(s) is $g'(x)$ negative if $g(x) = 3f(x)$?



(a) $(-\infty, A), (C, \infty)$

(b) $(-\infty, B), (D, \infty)$

(c) (A, C)

(d) (B, D) only

(e) none of these

30. Given $f(x) = \begin{cases} x+2 & \text{if } x < 2 \\ x^2 & \text{if } x = 2 \\ 5 & \text{if } x > 2 \end{cases}$. Which of the following statements is true?

(a) $f(x)$ is continuous from the left at $x = 2$.

(b) $f(x)$ is continuous from the right at $x = 2$.

(c) $f(x)$ is continuous at $x = 2$.

(d) None of these is true.

(e) $f(x)$ is not continuous at $x = 2$ because $\lim_{x \to 2} f(x)$ exists but does not equal $f(2)$.

31. Compute $\lim_{x \to -\infty} \frac{5e^{2x} - 8e^{-3x}}{3e^{2x} + 2e^{-3x}}$

(a) 0

(b) $\frac{5}{3}$

(c) -4

(d) $-\infty$

(e) ∞

32. Calculate $\lim_{x\to\infty} [\ln(1+2x) - \ln(2+x)]$.
   (a) 0
   (b) 1
   (c) $\ln(2)$
   (d) ∞
   (e) $-\infty$

33. The domain of $f(x)$ is all real numbers and $f''(x) = 3x(x^2-16)(x-4)$. Give the x-coordinate of the inflection point(s).
   (a) $x = 0, x = 4,$ and $x = -4$
   (b) $x = 0$ and $x = -4$ only
   (c) $x = 4$ and $x = -4$ only
   (d) $x = 0$ and $x = 4$ only
   (e) $f(x)$ has no inflection points.

34. An object is moving according to the equation of motion $s(t) = \cos t + \frac{1}{4}t^2$. Find the time(s) when the acceleration is zero for $0 \le t \le 2\pi$.
   (a) $t = \frac{\pi}{3}, \frac{2\pi}{3}$
   (b) $t = \frac{\pi}{6}, \frac{5\pi}{6}$
   (c) $t = \frac{4\pi}{3}, \frac{5\pi}{3}$
   (d) $t = \frac{7\pi}{6}, \frac{11\pi}{6}$
   (e) $t = \frac{\pi}{3}, \frac{5\pi}{3}$

35. Find the derivative of the function $f(x) = \arcsin\left(e^{4x}\right)$
   (a) $f'(x) = \dfrac{4e^{4x}}{\sqrt{1-e^{8x}}}$
   (b) $f'(x) = -\dfrac{4e^{4x}}{\sqrt{1-e^{8x}}}$
   (c) $f'(x) = \dfrac{4e^{4x}}{1+e^{8x}}$
   (d) $f'(x) = -\dfrac{4e^{4x}}{1+e^{8x}}$
   (e) $f'(x) = \dfrac{4e^{4x}}{1-e^{8x}}$