



## MATH 151- WEEK-IN-REVIEW 5

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### DERIVATIVES

#### Basic Derivative Rules

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln(a)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

#### Inverse Trig Derivatives

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \cot^{-1}(x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$$

#### Extra Derivative Rules

##### Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

##### Quotient Rule

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

##### Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$



1. Differentiate the following functions. You don't need to simplify.

(a)  $f(x) = x^7 + \sqrt[4]{x} - \frac{5}{x} + \tan(x) - \csc(x)$

(b)  $g(t) = (2t + 5)(3 - t)$

(c)  $G(x) = \left(\frac{1}{x^2} - \frac{7}{x^5}\right)(3x - 2)$



$$(d) h(y) = \frac{7}{y^4}$$

$$(e) y = \frac{\sqrt[4]{x^3} + x}{x^2}$$

$$(f) F(y) = \frac{x^2 - x}{4x + 5}$$

$$(g) q(t) = (2 - 3x + 5x^2)^{50}$$



$$(h) H(t) = \frac{x}{(x^3 - 7)^5}$$

$$(i) P(x) = e^{\tan(x)} - 3^{5x^2-1}$$

$$(j) f(x) = \ln(3x^2 + 5x - 1) + \log_4(3 - x)$$

$$(k) y = e^{t \sin^2(t)}$$

$$(l) r(x) = \arccos(7x + 2)$$



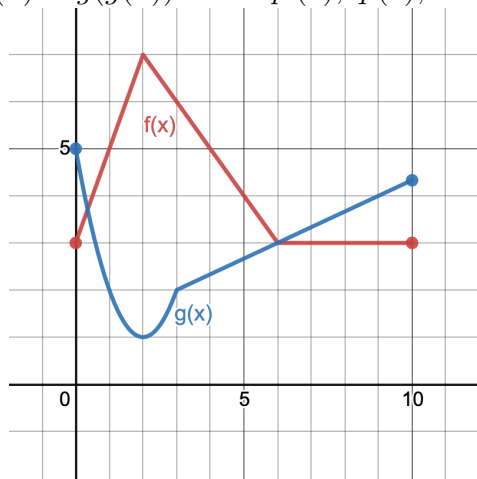
(m)  $g(y) = \csc(\tan(\cos(y)))$

2. Find the 2022<sup>nd</sup> derivative of  $y = 5 \sin(5x)$

3. Find the 2022<sup>nd</sup> derivative of  $y = xe^{2x}$



4. If  $f$  and  $g$  are the functions whose graphs are shown, let  $p(x) = f(g(x))$ ,  $q(x) = g(f(x))$ , and  $r(x) = g(g(x))$ . Find  $p'(2)$ ,  $q'(9)$ , and  $r'(8)$ .



5. Find  $\frac{dy}{dx}$  for each of the following.

(a)  $5x^2 - x^3y + 5y = 7$

(b)  $\sec(xy) = 3 - \cos(y)$



6. Find the equation of the tangent line to  $y = 3x + \sqrt{x}$  at  $(4, 14)$ .

7. Find the equation of the tangent line to  $x^2 + 6y^2 = 25$  at  $(1, 2)$ .

8. Given  $f(x) = \begin{cases} mx - b & \text{if } x < -1 \\ 5x^2 & \text{if } x \geq -1 \end{cases}$ . Find values for  $m$  and  $b$  that make the function differentiable everywhere.



9. For what values of  $a$  and  $b$  is the line  $y = 3x - 1$  tangent to the parabola  $y = ax^2$  when  $x = 4$ ?

10. Show there are two tangent lines to the parabola  $y = 4x^2$  that pass through the point  $(0, -4)$ . Find the equation of these tangent lines.