Math 151- Week-In-Review 8

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What does the derivative tell us?

1. Find the absolute maximum and minimum values of each of the following functions on the given interval.
   (a) $y = \dfrac{1}{x}$ on $[1, 5]$

   (b) $f(x) = -5x^3$ on $[-2, 4]$

(c) $g(x) = \begin{cases} x+5 & -4 < x \leq -1 \\ 3-x^2 & -1 < x < 3 \\ 5-x & 3 \leq x < 4 \end{cases}$

(d) $h(x) = x^2 e^{-x}$ on $[0, 4]$

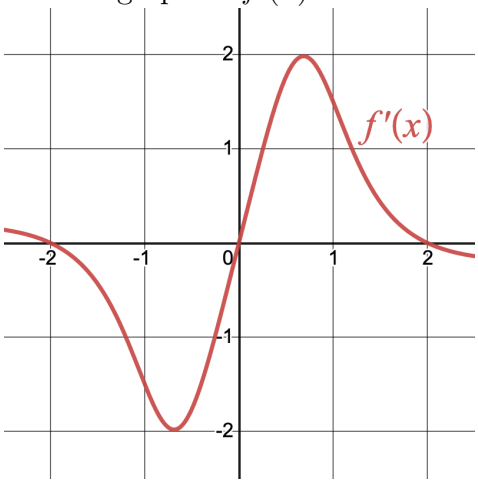
2. Find the critical values of the following functions.

(a) $f(x) = \dfrac{7-x}{x+1}$

(b) $g(x) = (x^3 - 12x)^{1/3}$

3. Classify the local extrema of $f(x)$ given $f'(x) = (x-3)^5(x+1)(x+7)^8$.

4. Given the graph of $f'(x)$ below find the given intervals/values.



(a) Intervals where $f(x)$ is increasing

(b) Intervals where $f(x)$ is decreasing

(c) x-values of any local maxima

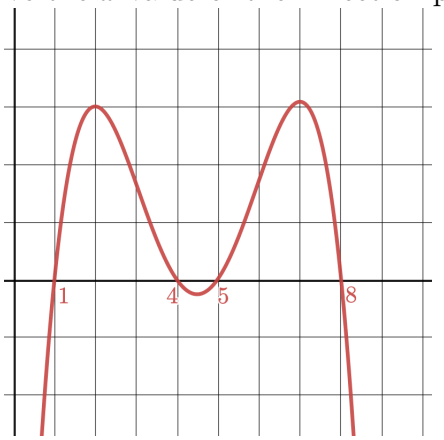
(d) x-values of any local minima

(e) Intervals where $f(x)$ is concave up

(f) Intervals where $f(x)$ is concave down

(g) x-values of any points of inflection

5. Give the x-value of the inflection points of f for each part.

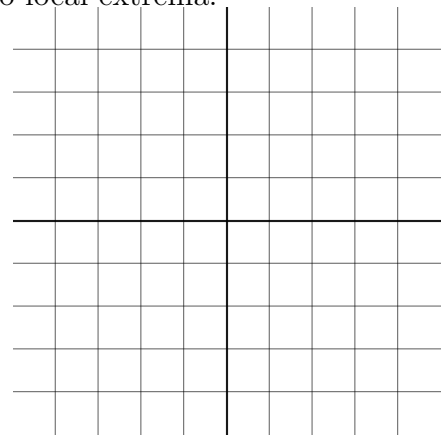


(a) The above curve is the graph of f.

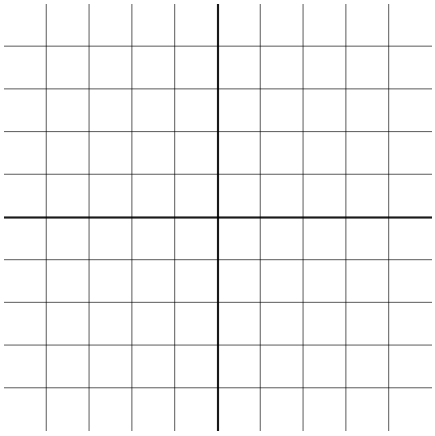
(b) The above curve is the graph of f'.

(c) The above curve is the graph of f''.

6. Sketch a graph of a continuous function where $x = -1$ is a critical number, but the function has no local extrema.



7. Sketch a graph of a continuous function where $x = -1$ is a local minimum and the function is not differentiable at $x = 3$.



8. If $f'(x) = x(4x-1)^{2/3}$, find where the function is concave up. Are there any points of inflection?

9. If $f(x) = x^2 \ln\left(\frac{x}{4}\right)$, find where the function is concave up. Are there any points of inflection?

10. Does $f(x) = x\sin x + \cos x$ satisfies the Mean Value Theorem on $[0, 2\pi]$, and find all c that satisfies the conclusion of the Mean Value Theorem.

11. Find the given limits.

(a) $\lim\limits_{x\to 0} \dfrac{\arccos(x) - \frac{\pi}{2}}{3x}$

(b) $\lim\limits_{x\to\infty} x^3 e^{-x^3}$

(c) $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{e^x - 1} \right)$

(d) $\lim_{x \to 0^+} (3x + 1)^{\csc x}$