Note ♯1 (The Substitution Rule, Areas Between Curves)

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[The Substitution Rule]

(1) Evaluate the integral.

(a) $\displaystyle\int \frac{\sin^{-1}(5x)}{\sqrt{1-25x^2}}\,dx$

(b) $\displaystyle\int 6e^{3x}\sin(e^{3x})\,dx$

(c) $\displaystyle\int \frac{1}{\cos^2(7x)\sqrt{3-\tan(7x)}}\,dx$

(d) $\displaystyle\int \frac{\cos\sqrt{3x-4}}{\sqrt{3x-4}}\,dx$

(e) $\displaystyle\int x^5 \sqrt[7]{x^3 + 11}\, dx$

(f) $\displaystyle\int \frac{\csc^2(\frac{1}{x^3})}{x^4}\, dx$

(g) $\displaystyle\int \frac{5x - x^3}{1 + x^4}\, dx$

[**Areas Between Curves**]

(2) Sketch the region enclosed by the curves $y = \sqrt{2x+6}$ and $y = x+3$, and then find the area of the region.

(3) Sketch the region enclosed by the curves $y = 2x^2 + 5$ and $y = 5x^2 - 7$.

(4) Find the area bounded by the curves $x = 2y^2 + 4y + 2$ and $x = y^2 + y + 12$.

(5) Find the area between the curves $y = -4\sin(\frac{x}{2})$ and $y = 4\sin(\frac{x}{2}) - 4$ for $0 \leq x \leq \pi$.

(6) Find the area between the curves $y = e^{x/2}$ and $y = 1 - 2x$ from $x = -1$ to $x = 2$.

(7) Find the area bounded by the curves $y = \frac{4}{x}$, $y = \frac{x^2}{2}$, and $y = \frac{1}{6}x + \frac{1}{3}$.

(8) Find the area between the curves $y = \ln x$, $x - |y| = -2$, $y = -2$, and $y = 2$.

(9) Find the area of the region bounded by the function $y = \sqrt{x-3}$, the tangent line to the function when $x = 7$, and the x-axis.