Note ♯10 (Exam3 Review)

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(1) Determine whether the following series converge or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{7+\arctan n}{n^3}$

(b) $\sum_{n=3}^{\infty} \frac{5+2\sin n}{n}$

(c) $\displaystyle\sum_{n=3}^{\infty} \frac{5n^3 - 2n^2}{n^4 + 5n^2 + 9}$

(d) $\sum_{n=0}^{\infty} \frac{n2^n}{3^{2n}}$

(2) Use the Alternating Series Estimation Theorem to estimate the error in using s_5 (the sum of the first five terms) to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

(3) Determine whether the following series absolutely convergence, conditionally convergence, or divergence.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(2n+1)!}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

(d) $\sum_{n=2}^{\infty} \frac{n^2-3}{n^4-n+1}$

(e) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n+7}$

(4) Consider two series with positive terms, $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$. If $\sum_{n=0}^{\infty} a_n$ diverges, which of the following is true?

(a) If $b_n > a_n$ for all n, then $\sum_{n=0}^{\infty} b_n$ diverges.

(b) If $b_n < a_n$ for all n, then $\sum_{n=0}^{\infty} b_n$ converges.

(c) If $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=0}^{\infty} b_n$ converges.

(d) If $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=0}^{\infty} b_n$ diverges.

(5) Suppose that the power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 3$ and diverges when $x = 6$. Which of the following statements is certain to be true?

(a) $\sum_{n=0}^{\infty} c_n(-6)^n$ is divergent.

(b) $\sum_{n=0}^{\infty} c_n(-4)^n$ is convergent.

(c) $\sum_{n=0}^{\infty} c_n(-3)^n$ is convergent.

(d) $\sum_{n=0}^{\infty} c_n(-2)^n$ is convergent.

(6) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n!(2x-3)^n}{5^n}$.

(7) Find the radius of convergence and the interval of convergence of the power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2x+1)^n}{n7^n}$

(b) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{5^n n!}$

(8) Find the power series of $f(x) = \dfrac{x^3}{8+x}$ centered at 0.

(9) Find the Maclaurin series.

(a) $f(x) = x\cos(x^2)$.

(b) $f(x) = \dfrac{x^2}{(1+5x)^2}$

(10) Find the Maclaurin series for $f(x) = x^4 \arctan(2x)$.

(11) Find the radius of convergence for the Maclaurin series representation of $f(x) = \dfrac{1}{1+9x^2}$.

(12) Find the third degree Taylor polynomial, $T_3(x)$, centered at $a = 1$ for the function $f(x) = \sqrt{x}$.

(13) Find the third degree Taylor polynomial, $T_3(x)$, centered at $a = -1$ for the function $f(x) = x^4 + 5x^3 + 4x^2 + x + 7$.

(14) Find the Taylor series for $f(x) = xe^x$ about $a = 5$.

(15) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+2}}{3^{2n+1}(2n+1)!}$.

(16) Find the sum of the series $\sum_{n=0}^{\infty} \frac{3^{n+1}}{2^n n!}$.

(17) Evaluate the integral as a power series.

(a) $\int x^3 \ln(1+7x^3)\, dx$

(b) $\int_0^1 x^2 \cos(5x^4)\, dx$