Note ♯11 (Final Review)

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(1) Calculate the area of the region bounded by the curves $4x + y^2 = 12$ and $x = y$.

(2) Calculate the area bounded by the curves $y = x^2 - 4$ and $y = -x^2 - 2x$ on the interval $-3 \leq x \leq 1$.

(3) Find the volume of the solid found by rotating the region bounded by the curves $y = -x^2 + 2x$ and $y = 0$ about the y-axis.

(4) Find the volume of the solid found by rotating the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$ about the line $x = 2$.

(5) Find the volume of the solid found by rotating the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$ about the line $y = -4$.

(6) The base of a solid is the region bounded by the curve $y = 5 - x^2$ and the x-axis. Cross-Sections perpendicular to the y-axis are rectangles with height equal to twice the base. Find the volume of this solid.

(7) A spring has a natural length of 2 m. If a force of 12 N is required to hold the spring to a length of 4 m, find the work done to stretch the sping from 3 m to 5 m.

(8) A heavy rope, 50 ft long and weighing 15 lbs, hangs over the edge of a building. How much work W is done in pulling the rope up 10 ft?

(9) Evaluate $\int \frac{1+x}{1+x^2}\, dx$.

(10) Evaluate $\displaystyle\int_0^1 \frac{4x^2+5}{2x+1}\,dx$.

(11) Write out the form of the partial fraction decomposition of the function

$$f(x) = \frac{x^3 - 2x^2 - 5x + 4}{(x+2)^2(x^2-1)(x^2+5x+7)}$$

(12) Evaluate $\int \dfrac{-2x+4}{(x^2+1)(x-1)}\,dx.$

(13) Evaluate $\int (3x + 14)e^{-2x}\, dx$.

(14) Evaluate $\int \sin^2 x\, dx$.

(15) Evaluate $\int \sin^7\theta\cos^5\theta\ d\theta$.

(16) Evaluate $\int \tan^3 x \sec^5 x \; dx$.

(17) Evaluate $\int \sqrt{9-x^2}\, dx$.

(18) Evaluate $\int \sqrt{4x^2 - 9}\, dx$.

(19) Evaluate $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$.

(20) Determine whether the following series converge or diverge. Support your answer.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(c) $\sum_{n=3}^{\infty} \frac{\ln n}{n}$

(d) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(21) Find the Radius of Convergence and Interval of Convergence of the power series $\sum_{n=2}^{\infty} \frac{(3x-1)^n}{8^n(n-1)}$.

(22) The series $\sum_{n=2}^{\infty} c_n x^n$ converges when $x = 4$ and diverges when $x = -7$. What can be said about the convergence of the following series?

$$\text{(I)} \sum_{n=2}^{\infty} c_n 9^n \qquad \text{(II)} \sum_{n=2}^{\infty} c_n (-4)^n$$

(a) Both (I) and (II) are inconclusive.
(b) (I) diverges, (II) converges.
(c) (I) diverges, (II) is inconclusive.
(d) Both (I) and (II) converge.
(e) (I) is inconclusive, (II) converges.

(23) Find the Radius of Convergence for the Maclaurin series representation of $f(x) = \dfrac{2x}{5-3x}$.

(24) Find the Maclaurin series for the function $f(x) = x^2 e^{-x^3}$.

(25) Find the Maclaurin series for the function $f(x) = \sin(2x)$.

(26) Evaluate the indefinite integral as a Maclaurin series.

$$\int x \arctan(4x^2)\, dx$$

(27) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{4^{2n}(2n)!}$.

(28) Find the sum of the series $\sum_{n=0}^{\infty} \frac{9(-1)^n \pi^{2n+1}}{2^{2n+1}(2n+1)!}$.

(29) Consider the following parametric equation

$$x = 6\cos\theta, \quad y = 7\sin\theta, \quad 0 \leq \theta \leq 2\pi.$$

Eliminate the parameter to find a Cartesian equation of the curve.

(30) Find the length of the curve

$$x = 6t - 2t^3, \quad y = 6t^2, \quad 0 \leq t \leq 3$$

(31) Find the length of the curve

$$x = e^t + e^{-t}, \quad y = 5 - 2t, \quad 0 \leq t \leq 2$$

(32) Find the length of the arc of the curve given by $x(t) = 3t - t^3$, $y(t) = 3t^2$ from $(0, 0)$ to $(-2, 12)$.

(33) Find the exact area of the surface obtained by rotating the given curve about the x-axis.

$$x = 6t - 2t^3, \quad y = 6t^2, \quad 0 \leq t \leq 2$$

(34) Find the Cartesian coordinate of the polar coordinate $(3, \pi)$.

(35) Find the Polar coordinate of the Cartesian coordinate $(4, -4)$.

(36) Find the Polar coordinate of the Cartesian coordinate $(-1, \sqrt{3})$.

(37) Find a Cartesian equation for the curve $r = 7\sin\theta$.

(38) Find the area of the region that is bounded by the given curve and lies in the specified sector.

$$r = 7\cos\theta, \quad 0 \leq \theta \leq \pi/6$$

(39) Find the area of the region that lies inside the first curve and outside the second curve.

$$r = 6 - 6\sin\theta, \quad r = 6$$

(40) Find the area of the region enclosed by one loop of the curve.

$$r = \sin(8\theta)$$

(41) Find the area of the region enclosed by on loop of the curve.

$$r = 5\cos(11\theta)$$