



NOTE #3 (VOLUMES BY CYLINDRICAL SHELLS, WORK)

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[Volumes by Cylindrical Shells]

- (1) Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x-1}$, $y = 1$, $y = 3$ and the y -axis around the x -axis.



- (2) Find the volume of the solid obtained by rotating the region bounded by $y = 3x - x^2$ and $y = 3x - 9$ around the line $x = 5$.



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- (3) Find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 3y + 4$ and $x = -y^2 + y + 4$ around the line $y = -1$.



- (4) Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $x = y^3$ around the line $x = -3$.



[Work]

- (5) A particle is moved along the x -axis by a force that measures $f(x) = x^3\sqrt{x^2 - 9}$ pounds at a point x feet from the origin. Find the work done in moving the particle from $x = 3$ to $x = 5$.



- (6) Suppose a spring has a natural length of $0.1m$ and it takes $8N$ of force to hold it stretched to $0.3m$. How much work is required to stretch it from $0.15m$ to $0.25m$?



- (7) Suppose it takes $10 \text{ ft} - lb$ to stretch a spring from 2 ft to 5 ft beyond its natural length.
- (a) How much work is require to stretch the spring from 1 ft to 4 ft beyond its natural length?
 - (b) How far beyond its natural length would a force of 2 lb keep the spring stretched?



- (8) A rope that is 30 m long and weighs 10 kg hangs over the edge of the building.
- (a) How much work is done pulling all the rope to the top?
 - (b) How much work is done pulling half the rope to the top?
 - (c) Is the work to pull the all the rope to the top double pulling half the rope to the top? Why or why not?



- (9) A 400 *lb* piano is lifted 36 *ft* to be placed into an apartment through a large window. The cable used to lift the piano weighs 1 *lb* for every 6 *ft*. How much work is done?



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- (10) A swimming pool is 10 m wide and 20 m long. The pool is 7 m deep at one end and slopes up the surface at the other end.
- (a) If the pool is full, how much work is done pumping all the water out of the pool?
 - (b) If the pool is full, how much work is done if the water is pumped out until 3 m of water is left in the deep end? Setup the integral only.
 - (c) If the pool was only filled till it is 5 m deep in the deep end, how much work is done to pump all the water out of the pool? Setup the integral only.